11.3 Angle modulation (FM/PM)

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS frequency deviation • Carson's rule

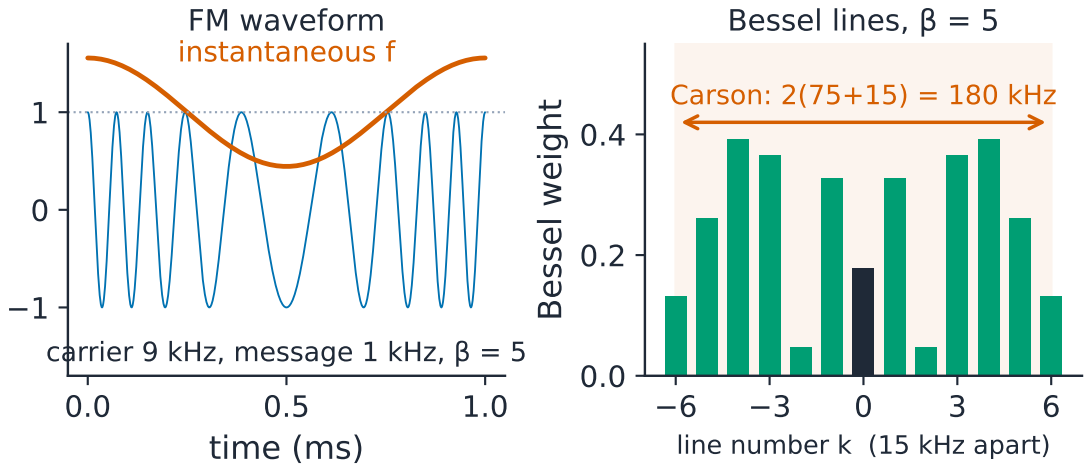
Objective

Derive instantaneous frequency and Carson bandwidth for FM and PM, tell narrowband from wideband, and explain capture and threshold qualitatively.

1 Key knowledge and worked examples

■ The two angles

FM: $f_i = f_c + k_f \cdot x(t)$, deviation Δf, $\beta = \Delta f / f_m$. PM: phase $k_p \cdot x(t)$, $\beta = k_p \cdot A_m$ – frequency-independent.



The message bends the instantaneous frequency; Carson's rule fences the spectral spread.

■ Carson's rule

$B \approx 2(\Delta f + f_m) = 2f_m(1+\beta)$: broadcast's Δf = 75 kHz, $f_m = 15kHz$ gives $\beta = 5$ and B = 180 kHz inside a 200 kHz channel ✓.

■ **Narrow vs wide**

$\beta \ll 1$: $B \approx 2f_m$ (narrowband, walkie-talkie). $\beta \gg 1$: B $\approx 2\Delta$f, and the post-detector SNR grows as $3\beta^2/2$ – bandwidth bought with signal-to-noise (unit 11.4).

2 Practice

2.1 Compute β and Carson bandwidth for Δf = 40 kHz, $f_m = 8kHz$. [3]

2.2 A narrowband system runs $\beta = 0.15$ at $f_m = 4kHz$. Estimate its bandwidth. [3]

2.3 A PM modulator has $k_p = 2.5rad/V$ and the message amplitude is 2 V. Compute β. [3]

3 Exam-style questions

3.1 An FM link uses Δf = 50 kHz with messages up to 15 kHz.

Compute β and Carson's bandwidth at the top message frequency. Compute B for a 300 Hz tone at the same deviation, and comment on the ratio. The channel is 150 kHz wide: state whether the design fits at both message frequencies. Explain why FM's bandwidth is set by the WORST (highest) modulating frequency. [8]

3.2 Compare FM and PM for a message whose amplitude is constant but whose frequency varies from 100 Hz to 10 kHz, with Δf (FM) = 40 kHz and k_p (PM) chosen to give $\beta = 4$ at 1 kHz.

Compute FM's β at 100 Hz and at 10 kHz. Compute PM's β at the same two frequencies. State which system keeps its deviation (and bandwidth) sensible across the range and which needs pre-emphasis, and explain the threshold/capture behaviour a wideband receiver shows near the FM threshold. [7]