11.1 Signals, spectra and bandwidth

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS bandwidth • sideband

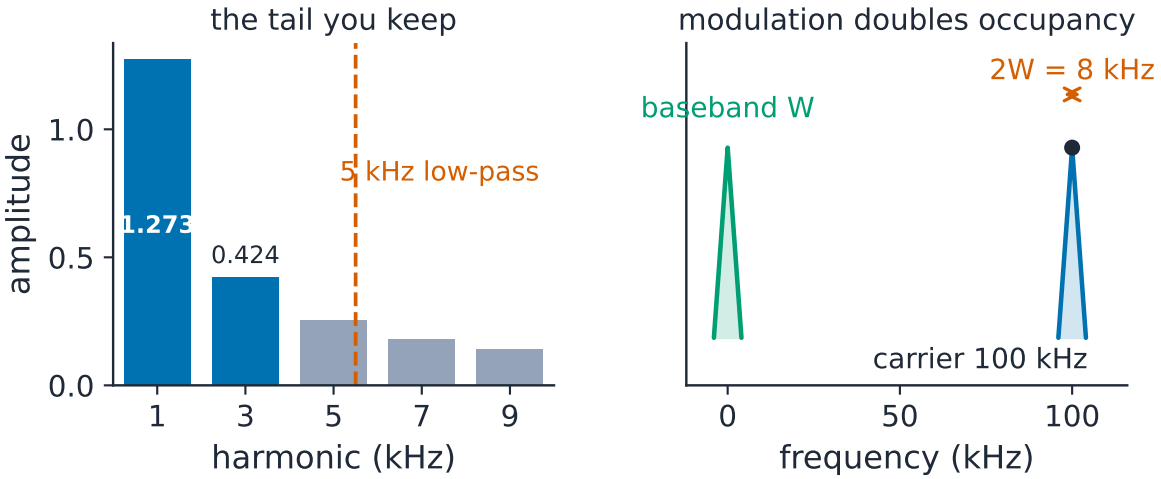
Objective

Compute the spectra of standard signals and modulated waveforms, compare bandwidth measures, and fit signals to channel constraints.

1 Key knowledge and worked examples

Harmonic structure

A square wave carries odd harmonics at $4/(\pi k)$ of the fundamental: 1st at 1.273, 3rd at 0.424, 5th at 0.255 – and 81% of the power sits in the fundamental ($8/\pi^2 = 0.811$), the Parseval intuition.



The tail you keep is the bandwidth you pay for; modulation doubles the occupancy.

Worked example: truncation

A 1 kHz square wave through a 5 kHz low-pass keeps the 1st and 3rd harmonics; the 5th (at 5 kHz, amplitude 0.255) is the boundary case – the reconstructed wave rounds over exactly as Fourier predicts.

■ **Modulation shifts, never reshapes**

A baseband signal of width W mixed onto a carrier occupies 2W of radio spectrum (both sidebands); SSB halves it by sending one. A 4 kHz message on a 100 kHz carrier: 96–104 kHz.

2 Practice

2.1 Compute the 3rd- and 5th-harmonic amplitudes (relative to the fundamental) of a square wave. [3]

2.2 A 2 kHz square wave passes a 4 kHz low-pass filter. Which harmonics survive, and what does the output look like? [3]

2.3 A telephone channel passes 300–3400 Hz. Compute its bandwidth and the minimum sampling rate Nyquist demands. [3]

3 Exam-style questions

3.1 A 1 kHz square wave feeds a low-pass channel.

Give the amplitudes of the first three odd harmonics (relative to a unit fundamental). Compute what fraction of the total power the fundamental alone carries, stating the series identity you use. The channel cuts at 4 kHz: state which harmonics pass and sketch-describe the output waveform. Explain why "bandwidth" for this signal is a design choice rather than a single number. [8]

3.2 A message occupying 0–4 kHz amplitude-modulates a 100 kHz carrier (conventional double-sideband AM).

Draw/describe the spectrum: give the frequency ranges of the upper and lower sidebands and the total occupied bandwidth. Repeat for single-sideband (upper sideband only). If the channel assignment is exactly 8 kHz wide, state which scheme fits and what it costs. [7]