10.7 PID control and compensation

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS PID controller PID • lead compensator

Objective

Tune PID terms against measured specs, size lead compensators in the frequency domain, and handle saturation with anti-windup.

1 Key knowledge and worked examples

■ The three terms

$C = K_p + K_i/s + K_d s$: P reacts to present error, I to its history (kills steady-state error, adds a type), D to its slope (adds phase, dies in noise). Saturation invalidates the linear story – the integrator winds up unless clamped or back-calculated.

■ Worked example: Ziegler–Nichols from the edge

Plant $1/[s(s+1)(s+2)]$: Routh gives $K_u = 6$ (margin at $\omega_u = \sqrt{2} = 1.41$), so $T_u = 2\pi/1.41 = 4.44s$. ZN PID: $K_p = 0.6K_u = 3.6$, $T_i = T_u/2 = 2.22s$, $T_d = T_u/8 = 0.555s$ – and the closed-loop characteristic computes to $s^4 + 3s^3 + 4.0s^2 + 3.6s + 1.62$ with all four roots in the LHP ✓.

■ Lead sizing

$(1+\tau s)/(1+\alpha\tau s)$, $0 < \alpha < 1$: max phase $sin\varphi_m = (1-\alpha)/(1+\alpha)$ at $\omega_m = 1/(\tau\sqrt{\alpha})$. For $+40°$: $\alpha = 0.217$; high-frequency gain rises by $1/\alpha = 4.6$ – the price the rest of the design pays.

2 Practice

2.1 A plant oscillates at $K_u = 8$, $T_u = 1.5s$. Give the ZN settings for P and for PI. [3]

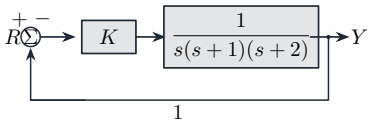
2.2 Compute α for a lead network aiming at $+40°$ maximum boost. [3]

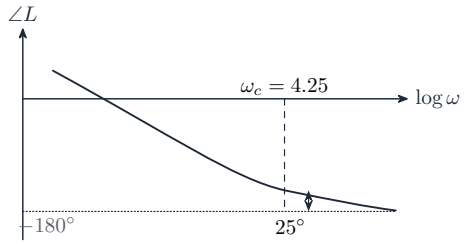
2.3 A P-only loop $K_p = 5$ around $4/[(s+1)(s+2)]$: compute the unit-step steady-state error, and state what adding integral action does to it. [3]

3 Exam-style questions

3.1 A unity-feedback loop around the plant $G = 1/[s(s+1)(s+2)]$ is to be tuned by Ziegler–Nichols.

Show from Routh that proportional gain first loses stability at $K_u = 6$, and give the oscillation frequency. Compute T_u and the full Ziegler–Nichols PID settings. Explain what each of the three terms contributes here (error type, phase effect), and why ZN settings are a starting point rather than a final tuning. [8]



3.2 A loop $L = 20/[s(s+2)]$ has PM $= 25°$ ($\omega_c = 4.25$ rad/s) and must reach at least $45°$.

Compute the phase boost needed at the new crossover and add margin for the crossover shift (design the lead for $+30°$). Size α and state the gain the network adds at high frequency. Choose τ so the boost lands at $\omega_m \approx 4.5 rad/s$, giving the compensator's zero and pole. State the remaining step of the design (re-locate the crossover) and why. [7]