

10.6 Bode and Nyquist analysis

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS gain margin • phase margin • Nyquist criterion

Objective

Draw Bode plots, read gain and phase margins off them, apply the Nyquist criterion with correct encirclement counting, and connect margins to closed-loop ringing.

1 Key knowledge and worked examples

■ Margins

Gain margin: how many times the gain can rise before $|L| = 1$ at the -180° crossing.

Phase margin: $PM = 180^\circ + \angle L(j\omega_c)$ at the gain crossover. Working rule: $\zeta \approx PM/100$ – $PM\ 50^\circ \approx 16\%$ overshoot.

■ Worked example: PM by formula

$L = 25/[s(s+5)]$: crossover solves $\omega\sqrt{\omega^2+25} = 25 \rightarrow \omega_c = 3.93$; phase $-90^\circ - \text{atan}(3.93/5) = -128.2^\circ \rightarrow PM = 51.8^\circ$ – a well-damped design by two lines of arithmetic.

■ Nyquist in one number

$L = K/[s(s+2)(s+4)]$ crosses -180° at $\omega = \sqrt{8} = 2.83$ where $|L| = K/48$: the -1 encirclement (and stability) flips at $K = 48$ – exactly Routh's answer for $s^3 + 6s^2 + 8s + K$.

2 Practice

2.1 Evaluate magnitude (dB) and phase of $10/(s+1)$ at $\omega = 1$ rad/s. [3]

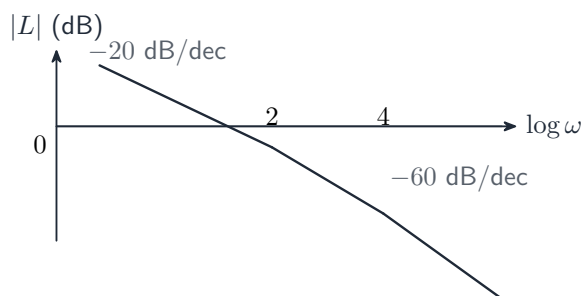
2.2 Compute PM for $L = 25/[s(s+5)]$. [3]

2.3 For $K = 12$ in $L = K/[s(s+2)(s+4)]$, compute GM (as a factor and in dB). [3]

3 Exam-style questions

3.1 A loop $L = 24/[s(s+2)(s+4)]$ (unity feedback).

Compute the gain crossover frequency (solve $\omega\sqrt{(\omega^2+4)(\omega^2+16)} = 24$). Compute the phase margin there. Compute GM from the -180° crossing. Predict ζ and the overshoot from the PM using the rule, and check the trend against the Routh stability bound ($K < 48$). [8]



3.2 A plant has open loop $L = K(s+1)/[s(s-1)(s+4)]$ – note the pole at $+1$.

State P (open-loop RHP poles). Write the characteristic polynomial and use Routh to find the range of K for closed-loop stability. State what the Nyquist criterion then REQUIRES the plot to do (how many encirclements, which way) for stability in that range, and what it does for K just below the boundary. [7]

