

10.5 Root locus

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS root locus • breakaway point

Objective

Sketch root loci with the standard rules, design gain against a damping specification, and predict where closed-loop poles migrate.

1 Key knowledge and worked examples

■ The rules that build the sketch

Locus exists left of an odd count of real poles+zeros; $n-m$ branches go to asymptotes $180^\circ(2k+1)/(n-m)$ centred on the centroid $\sigma = (\sum p - \sum z)/(n-m)$; branches meet at the breakaway points where $\frac{d}{ds}[\text{denominator/numerator}] = 0$.

■ Worked example: the three-pole plant

$K/[s(s+4)(s+6)]$: centroid -3.33 , asymptotes $\pm 60^\circ/180^\circ$; breakaways at -1.57 ($K = 16.9$) and -5.10 ; Routh says stable to $K = 240$ – the locus crosses into the RHP at $\omega = \sqrt{24} = 4.90 \text{ rad/s}$ at exactly that gain.

■ Design on a damping ray

For $\zeta = 0.5$ the pole must lie on $s = -0.5\omega + j0.866\omega$ where the angle condition holds; on $K/[s(s+4)(s+6)]$ the point is $-1.20 + j2.08$ with $K = 43.8$ – read the transient specs straight off the point.

2 Practice

2.1 Sketch facts for $K/[(s+1)(s+3)]$: give the asymptote angles, the centroid, and the breakaway point with its K . [3]

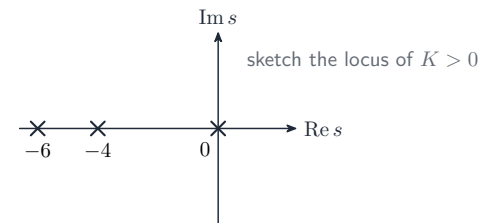
2.2 For $K(s+3)/(s+1)$: where does the second branch go (there is one zero at -3), and does a breakaway point exist on the real axis? [3]

2.3 For $K/[s(s+3)]$, verify that $s = -1.50 + j1.53$ satisfies the angle condition for $\zeta = 0.7$, and compute the K that places a closed-loop pole there. [3]

3 Exam-style questions

3.1 A unity-feedback system has open loop $K/[s(s+4)(s+6)]$.

Draw the real-axis segments that carry locus and compute the centroid and asymptote angles. Compute both breakaway points and the gain at the one between 0 and -4 . Use Routh to find the gain and frequency at which the locus crosses the imaginary axis. Assemble the sketch order: which specification (damping or speed) does moving along the locus from the breakaway towards the asymptotes trade against which? [8]



3.2 The plant $K/[s(s+6)]$ is stable for all K but poorly damped at high gain. Consider instead adding a zero – control $C(s)G(s) = K(s+2)/[s(s+6)]$ under unity feedback.

Write the closed-loop characteristic polynomial with unity feedback and show it is stable for ALL $K > 0$ (Routh first column). Compute the closed-loop poles at $K = 6$. Explain, in one sentence each, what the added zero did to the far branch and to the near branch. [7]
