10.3 Time-domain response

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS overshoot • settling time • steady-state error

Objective

Compute step-response specifications from the second-order prototype, map pole locations to transient behaviour, and read steady-state error constants off the loop.

1 Key knowledge and worked examples

■ The prototype formulas

$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, $t_p = \pi/(\omega_n\sqrt{1-\zeta^2})$, $t_s \approx 4/(\zeta\omega_n)$ (2% band). Poles at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$: real part decays, imaginary part rings, distance from origin is ω_n.

■ Worked example: one number, three specs

$\omega_n = 6$, $\zeta = 0.4$: $M_p = 25.4\%$, $t_p = 0.571s$, $t_s = 4/2.4 = 1.67s$ – and poles at $-2.4 \pm j5.51$ (check: $|pole| = \sqrt{2.4^2 + 5.51^2} = 6$ ✓).

■ Error constants

Type 0: $K_p = G(0)$, step error $1/(1 + K_p)$. Type 1: $K_v = \lim_{s\to 0} sG$, ramp error $1/K_v$; step error zero. Each integrator buys one order of tracking and costs phase.

2 Practice

2.1 A system has $\omega_n = 6rad/s$, $\zeta = 0.4$. Compute M_p, t_p and t_s (2%). [3]

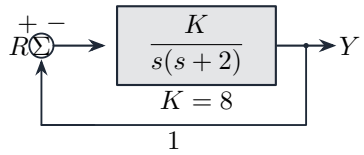
2.2 Poles at $-2 \pm j4$: compute ω_n, ζ and state whether the step response overshoots. [3]

2.3 Compute the dc gain and the unit-step steady-state error for $G = 5/(s+2)$ in unity feedback. [3]

3 Exam-style questions

3.1 A unity-feedback system has loop $G = K/[s(s+2)]$ with K = 8.

Write the closed-loop transfer function and its ω_n and ζ. Compute M_p, t_p and t_s. Compute the unit-ramp steady-state error via K_v. Determine the K that makes $\zeta = 0.5$ exactly, and verify. [8]



3.2 A servo must track a ramp with error at most 0.1 and settle (2%) within 2 s. The plant is $G = K/[s(s+2)]$.

Derive the ramp-error requirement on K_v and hence the minimum K. Derive the settling-time requirement on $\zeta\omega_n$ and check it at that K. If the two requirements conflict, state which gain each wants and propose the standard resolution (a compensator, not a compromise gain). [7]