

10.1 Modelling of dynamic systems

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS linearization • operating point

Objective

Derive first-order models from electrical, mechanical and thermal systems, linearize about an operating point, and read the physical meaning of the states.

1 Key knowledge and worked examples

■ One skeleton, every domain

Storage, flow, dissipation: $C \cdot dv/dt = (v_{in} - v)/R$ (RC), $J \cdot d\omega/dt = T - B\omega$ (rotor), $C \cdot dT/dt = P - hT$ (thermal). Each gives a time constant and a final value with no differential-equation machinery.

■ Worked example: the DC motor

Torque $K \cdot i$, back-EMF $K \cdot \omega$, $J = 0.01$, $B = 0.1$, $R = 1$, L negligible: $\omega/V = K/[(Js + B)R + K^2] = 50/(s + 35) - \tau = 28.6$ ms, 1.43 rad/s per volt.

■ Worked example: linearize the square root

Tank outflow $q = k\sqrt{h}$ at $\bar{h} = 4$, $k = 2$: $\delta q \approx (k/(2\sqrt{\bar{h}})) \cdot \delta h = 0.5 \cdot \delta h$; with $A = 1$ m², $\delta h' = \delta q_{in} - 0.5\delta h$ – a first-order tank with $\tau = 2$ s. The small-signal gain is the slope at the operating point.

2 Practice

2.1 An RC network has $R = 2$ k Ω , $C = 100$ μ F. Compute the time constant and the final capacitor voltage for a 9 V step. [3]

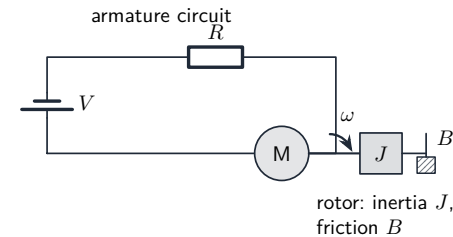
2.2 A thermal system: $C = 200$ J/K, $h = 4$ W/K, $P = 100$ W. Compute the time constant and the final temperature rise. [3]

2.3 A pendulum of length 0.4 m is linearised about straight-down. Compute its natural frequency in rad/s. [3]

3 Exam-style questions

3.1 An armature-controlled DC motor has $J = 0.02$ kg $\cdot$ m², $B = 0.2$ N $\cdot$ m $\cdot$ s, $K = 0.4$ V $\cdot$ s/rad, $R = 2$ Ω (inductance negligible). The input is armature voltage V , the output is speed ω .

Write the two first-order equations (electrical and mechanical, with the back-EMF coupling). Eliminate the current to find $\omega(s)/V(s)$ as a single first-order transfer function. Compute the time constant and the steady state speed per volt. State the physical meaning and units of both state variables (i , ω). [8]



3.2 A mixing tank of area 1 m^2 drains through a valve with outflow $q_{\text{out}} = 2\sqrt{h}$ (m^3/h with h in m) about the operating point $\bar{h} = 4 \text{ m}$; the inflow is q_{in} .

Linearise the outflow about $\bar{h}$ and write the small-signal model for δh . Compute the linearised resistance (slope of q vs h) and the tank's time constant. Compute the new time constant if the operating point moves to $\bar{h} = 9 \text{ m}$, and explain physically why a deeper tank responds more slowly. [7]

