# 1.7 First-order RC and RL response

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

**KEY WORDS** natural response • step response • time constant • flyback diode

## Objective

Write first-order RC and RL responses directly from initial value, final value and time constant; solve switching problems; and explain – with numbers – the hazards of interrupting inductor current.

## 1 Key knowledge and worked examples

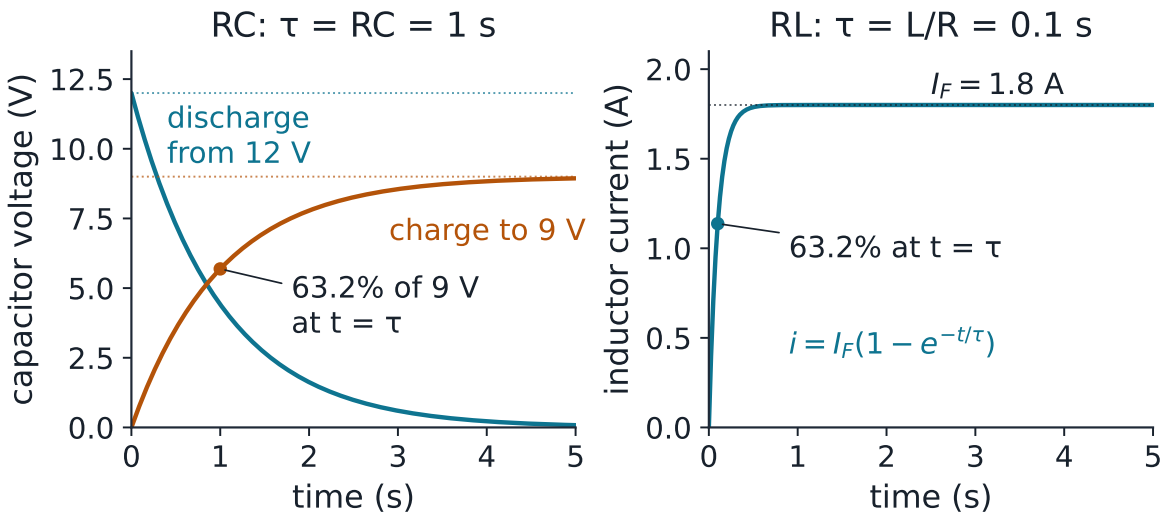
### ■ One formula for every first-order circuit

Every circuit with one storage element answers to

$$v(t) = V_F + (V_0 - V_F)\, e^{-t/\tau}$$

$V_0$ comes from the stored energy before the switch (voltage continuity for C, current continuity for L); $V_F$ comes from the circuit after the switch, with the element replaced by its steady-state open (C) or short (L); $\tau = RC$ or $\tau = L/R$ uses the resistance the element sees with sources zeroed.

At $t = \tau$ the gap has closed to 37%; after $5\tau$ the circuit is within 1% and treated as settled.




■ **Worked example: discharge and charge with the same $\tau$**

A 100 µF capacitor charged to 12 V discharges through 10 kΩ: $\tau = 1s$, so $v(t) = 12e^{-t}$ V. After 2 s: $12e^{-2} \approx 1.62V$.

Now switch the same RC to a 9 V source from 0 V: $V_0 = 0$, $V_F = 9$, so $v(t) = 9(1-e^{-t})$; at $t = 1s$ it reaches $9 \times 0.632 \approx 5.7V$. The exponent's argument is dimensionless because $\tau$ carries the seconds.

■ **Worked example: RL rise and the switch-off spike**

$R = 8\Omega$, $L = 0.4H$ across a 12 V source: $I_F = 1.5A$, $\tau = L/R = 50ms$, $i(t) = 1.5(1 - e^{-t/0.05})$ A; at 100 ms, $1.5 \times 0.865 \approx 1.30A$.

Opening the switch forces the 1.5 A to stop in, say, 1 ms through a 0.1 H winding: $v = Ldi/dt = 0.1 \times 1500 = 150V$ – tens of times the supply, and the arc across the opening contacts is that voltage made visible. A **flyback diode** gives the current a path so `di/dt` stays small.

## 2 Practice

**2.1** Compute the time constant of 22 kΩ with 470 nF, and the settling time to within 1%. [3]

**2.2** An RL circuit has $R = 8\Omega$, $L = 0.4H$ and a 12 V source. Give $I_F$, $\tau$, and the current at $t = 0.1s$ starting from zero. [4]

**2.3** A 5 V source, 1 kΩ and 100 µF start with the capacitor at 1 V. Give $V_0$, $V_F$, $\tau$ and `v` at $t = 0.1s$. [4]

## 3 Exam-style questions

**3.1** A 9 V source charges 100 µF through 10 kΩ until fully charged; the capacitor is then switched to discharge through 20 kΩ.

Write both responses `v(t)` with their time constants on the same time axis (reset t = 0 at the switch). Then compute the time after the switch for the voltage to fall below 0.5 V, and verify the fully-charged claim using the $5\tau$ rule for the charging phase. [7]

**3.2** A relay coil models as 0.1 H with 20 Ω of resistance, driven from 12 V. The driver transistor interrupts the 0.6 A coil current in about 1 ms when switching off.

Compute the steady current, the coil's time constant, and the voltage the coil generates across the opening transistor if nothing limits it. Explain how a flyback diode changes this, and give the diode's required reverse rating. [6]