

9.6 Per-unit and power flow

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS power flow

Objective

Solve the two-bus power flow by hand: iterate to the receiving voltage, split the drop into its R and X parts, and size the compensation that restores it.

1 Key knowledge and worked examples

■ The two-bus update

Sending bus $1.0\angle 0^\circ pu$, line $Z = R + jX$, load $S = P + jQ$ at the receiving end. Iterate $V_2 \leftarrow 1.0 - Z \cdot \text{conj}(S/V_2)$ until it settles – the hand-scale Gauss-Seidel step.

■ The voltage-drop approximation

$\Delta V \approx (P \cdot R + Q \cdot X)/|V|$ – with network $X \gg R$, the REACTIVE flow through X dominates the drop, which is why shunt compensation is a voltage tool.

■ Worked example: compensation sweeps the curve

$Z = 0.02 + j0.10 pu$, load $0.8 + j0.4$: uncompensated $|V_2| = 0.937$. Adding $Q_c = 0.2$ at the load bus lifts it to 0.959; $Q_c = 0.4$ (full local Q supply) reaches 0.980 – and line loss falls from 0.0182 to 0.0133 pu because the current falls.

2 Practice

2.1 With $Z = 0.02 + j0.10 pu$ and load $0.8 + j0.4$, compute the approximate drop ΔV and the $P \cdot R$ and $Q \cdot X$ parts separately. [3]

2.2 The converged solution has line current 0.954 pu. Compute the line loss and the sending-end real power. [3]

2.3 For the same system with $Q_c = 0.2$ and $Q_c = 0.4$, compute $|V_2|$ (converged) and the corresponding line losses. [3]

3 Exam-style questions

3.1 A two-bus system: sending bus at 1.0 pu, $Z = 0.05 + j0.15pu$, receiving load $0.5 + j0.25$ pu.

Perform the first TWO Gauss–Seidel iterations from a flat start ($V_2 = 1.0$) and give each result in rectangular form. Iterate to convergence and state $|V_2|$ and its angle. Compute the line current, the real line loss and the sending-end real power, and verify the power balance. [8]

3.2 The system of the worked example ($Z = 0.02 + j0.10pu$, load $0.8 + j0.4$ pu) suffers a 0.937 pu receiving voltage.

Size the shunt compensation Q_c needed to restore $|V_2|$ to about 0.98, showing the sweep you used. Compute the percentage reduction in line loss at that compensation. Explain, from the drop decomposition, why a Q-based fix works better than a P-based one, and what device supplies Q_c in practice. [7]

Solutions

2.1

- $\Delta V \approx (0.8 \times 0.02 + 0.4 \times 0.10)/1.0 = 0.056pu$.
- Parts: $P \cdot R = 0.016$, $Q \cdot X = 0.040$ – the reactive part is 71% of the drop.

2.2

- Loss = $I^2 R = 0.954^2 \times 0.02 = 0.0182pu$.
- $P_s = 0.8 + 0.0182 = 0.818pu$.

2.3

- $Q_c = 0.2$: $|V_2| = 0.959$, loss 0.0148.
- $Q_c = 0.4$: $|V_2| = 0.980$, loss 0.0133.

3.1

- Iteration 1: $V_2 = 1.0 - (0.05 + j0.15)(0.5 - j0.25) = 0.9375 - j0.0625$.
- Iteration 2: $V_2 = 1.0 - (0.05 + j0.15) \cdot conj(S/V_2)$ with V_2 from step 1 $\rightarrow 0.9292 - j0.0619$.
- Converged: $|V_2| = 0.9306 \angle -3.85^\circ$.
- $I = conj(S/V_2) = 0.6007pu$.
- Loss = $I^2 R = 0.6007^2 \times 0.05 = 0.0180 pu$; $P_s = 0.5 + 0.0180 = 0.518 pu$.
- Balance: sending P minus loss equals load P ($0.518 - 0.018 = 0.500$) ✓; likewise $Q_s = 0.25 + I^2 X = 0.25 + 0.0541 = 0.304 pu$ covers load plus line consumption.

3.2

- Sweep: $Q_c = 0 \rightarrow 0.937$; $0.2 \rightarrow 0.959$; $0.4 \rightarrow 0.980$ – so $Q_c = 0.4pu$ (the load's full Q; 40 MVar on a 100 MVA base) reaches ~0.98.
- Loss falls $0.0182 \rightarrow 0.0133 pu = 27\%$ reduction.
- The drop decomposes as $P \cdot R + Q \cdot X = 0.016 + 0.040$: removing ALL of Q kills the 0.040 term, but removing the same pu of P would cut only the 0.016 term and shed revenue load as well.
- In practice a capacitor bank (or a STATCOM/SVC for fast duty) supplies Q_c at the bus, so the line no longer has to carry it.