

## 9.4 Synchronous machines

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 24 marks**

**KEY WORDS** power angle

### Objective

Fix speed by frequency, generate EMF by the flux rule, and trade real power against reactive power through the power-angle relation.

### 1 Key knowledge and worked examples

#### ■ Speed, EMF and the power-angle relation

Speed is exactly  $n_s = 120f/p$  – no slip. The EMF matches the transformer form  $E = 4.44fN\Phi k_w$ . For a round rotor on an infinite bus:  $P = 3VE/X_s \cdot \sin\delta$ , where  $\delta$  is the **power angle**.

#### ■ Worked example: the EMF

$f = 50\text{Hz}$ ,  $N = 100$  turns per phase,  $\Phi = 0.05\text{Wb}$ ,  $k_w = 0.9$ :  $E = 4.44 \times 50 \times 100 \times 0.05 \times 0.9 = 999\text{V}$  per phase – the same flux-frequency balance a transformer designer meets.

#### ■ Worked example: margin arithmetic

$V = 6.6\text{ kV (line)}$ ,  $X_s = 4\Omega$ ,  $E = 7.5\text{ kV (line)}$ : per phase  $3810.5\text{ V}$  and  $4330.1\text{ V} \rightarrow P_{max} = 3VE/X_s = 12.375\text{MW}$ . At  $\delta = 25^\circ$ :  $P = 5.23\text{MW}$ , static margin  $(12.375 - 5.23)/5.23 = 136\%$  – the arithmetic a grid operator runs before every dispatch.

### 2 Practice

**2.1** Compute the phase EMF of a 50 Hz machine with 100 turns per phase,  $\Phi = 0.05\text{Wb}$ ,  $k_w = 0.9$ . [3]

**2.2** Compute the synchronous speed of a 10-pole, 60 Hz machine, and state what happens to that speed when the load doubles. [3]

**2.3** With  $V = 6.6\text{ kV (line)}$ ,  $X_s = 4\Omega$ ,  $E = 7.5\text{ kV (line)}$ , compute the power at  $\delta = 25^\circ$  and the static margin. [3]

### 3 Exam-style questions

**3.1** A synchronous generator feeds an 11 kV (line) infinite bus through  $X_s = 5\Omega$  per phase, with  $E = 13\text{ kV (line)}$  and real output 10 MW.

Compute the maximum power and the operating power angle  $\delta$ . Compute the reactive power  $Q$  and state its sign and what it says about excitation. Compute the static margin and explain why operators keep it large. [8]

**3.2** A factory draws 500 kW at 0.8 power factor lagging. A synchronous motor rated 100 kW mechanical output is installed alongside it, and its excitation is adjusted until the overall factory power factor is exactly unity.

Compute the reactive power the factory draws. Compute the reactive power the motor must supply and the motor's kVA and power factor. Explain, in terms of excitation, how the machine does this while still delivering mechanical power. [7]

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## Solutions

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### 2.1

- $E = 4.44 \times 50 \times 100 \times 0.05 \times 0.9 = 999V$ .

### 2.2

- $n_s = 120 \times 60/10 = 720rpm$ .
- Nothing – speed is locked to the bus frequency; extra load only increases  $\delta$ .

### 2.3

- Per phase  $V = 3810.5$  V,  $E = 4330.1$  V:  $P = 5.23MW$ .
- Margin  $(12.375 - 5.23)/5.23 = 136\%$ .

### 3.1

- Per phase  $V = 6350.9$  V,  $E = 7507.8$  V:  $P_{max} = 3VE/X_s = 28.60MW$ .
- $\sin\delta = 10/28.60 \rightarrow \delta = 20.5^\circ$ .
- $Q = 3V(E \cos\delta - V)/X_s = 3 \times 6350.9 \times (7031.8 - 6350.9)/5 = +2.59$  MVar.
- Positive Q exported: the machine is over-excited and behaves reactively like a capacitor to the grid.
- Margin  $(28.60 - 10)/10 = 186\%$  – a large margin means a load swing or fault cannot push  $\delta$  past  $90^\circ$ , where synchronism is lost.

### 3.2

- $\tan\varphi = \tan(\cos^{-1}0.8) = 0.75$ ;  $Q = 500 \times 0.75 = 375kVar$ .
- Unity overall  $\rightarrow$  the motor supplies all of it:  $Q_{motor} = 375kVar$  (leading).
- $S_{motor} = \sqrt{100^2 + 375^2} = 388kVA$  at leading power factor  $100/388 = 0.258$ .
- Over-exciting the motor raises its E above the bus voltage, making it export reactive power – the power-angle relation carries the 100 kW at  $\delta \approx \sin^{-1}(100 \text{ kW} \cdot X_s/3VE)$  while  $Q = 3V(E \cos\delta - V)/X_s > 0$  supplies the compensation; the same machine is both motor and reactive source, which induction motors cannot do.