

Total: 25 marks

Objective

1 Key knowledge and worked examples

$n_s = 120f/p$ (rpm); $s = (n_s - n)/n_s$; rotor frequency $f_r = s \cdot f$. The rotor resistance appears as R'_s/s – slip turns the machine's behaviour into one element.

The air-gap power $P_{ag} = 3I^2 R'_2 / s$ gives the torque $T = P_{ag} / \omega_s$ (with $\omega_s = 2\pi n_s / 60$), and $P_{mech} = (1 - s)P_{ag}$; the rotor copper loss is exactly $s \cdot P_{ag}$ and the stator copper loss is $3I^2 R_1$. The balance $P_{in} = P_{mech} + s \cdot P_{ag} + 3I^2 R_1$ must close.

Neglecting stator impedance, peak torque comes at $s_{max} = R'_2/X$ – so a wound rotor that adds external resistance moves the peak toward start. With $R'_2 = 0.3\Omega$ and total reactance $X = 2.0\Omega$, adding $R_{ext} = 1.7\Omega$ puts the peak at $s = 1$: starting torque rises from $48.9 \text{ N} \cdot \text{m}$ to $176.8 \text{ N} \cdot \text{m}$ – bought with higher slip (and rotor loss) when running.

2 Practice

[3]

3]

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3 Exam-style questions

3]

3.2 The motor above is rebuilt as a wound-rotor machine. The total series reactance is $X = 2.0\Omega$ and $R'_2 = 0.3\Omega$.

Derive the condition on R'_2/s for maximum torque (stator resistance neglected), and hence the external rotor resistance that places maximum torque at starting ($s = 1$). Compute the starting torque without and with the added resistance. Explain the running penalty of leaving that resistance in circuit. [8]

Solutions

2.1

- $n_s = 120 \times 50/6 = 1000 \text{rpm}$.
- $s = (1000 - 970)/1000 = 0.03$.
- $f_r = 0.03 \times 50 = 1.5 \text{Hz}$.

2.2

- $R'_2/s = 0.5/0.03 = 16.7\Omega$.
- $16.7/0.5 = 33.3\times$ – slip transforms a small resistance into the dominant circuit element.

2.3

- $P_{ag} = 3 \times 20^2 \times (0.4/0.05) = 9600 \text{W}$.
- $\omega_s = 2\pi \times 1500/60 = 157.1 \text{rad/s}$; $T = 9600/157.1 = 61.1 \text{N} \cdot \text{m}$.
- $P_{mech} = (1 - 0.05) \times 9600 = 9120 \text{W}$.

3.1

- $Z = 0.4 + j1.6 + 0.3/0.04 = 7.9 + j2.4\Omega$; $|Z| = 8.26\Omega$, $I = (400/\sqrt{3})/8.26 = 28.0 \text{A}$.
- $P_{ag} = 3 \times 28.0^2 \times 7.5 = 17603 \text{W}$.
- $\omega_s = 157.1 \text{rad/s}$; $T = 17603/157.1 = 112.1 \text{N} \cdot \text{m}$.
- $P_{mech} = 0.96 \times 17603 = 16899 \text{W}$.
- Stator copper $3I^2R_1 = 939 \text{W}$; rotor copper $s \cdot P_{ag} = 704 \text{W}$. Balance: $P_{in} = 18542 \text{W} = 16899 + 704 + 939$ ✓ closes.
- $f_r = 0.04 \times 50 = 2 \text{Hz}$ – the rotor iron sees a 2 Hz flux, so its laminations need little thickness to keep eddy loss small.

3.2

- Torque $\propto P_{ag}$ peaks where $R'_2/s = X$ (maximum power transfer onto the rotor branch), i.e. $s_{max} = R'_2/X$.
- At start $s = 1$ requires $R'_2 + R_{ext} = X = 2.0\Omega \rightarrow R_{ext} = 1.7\Omega$.
- Without: $T_{start} = 3V^2R'_2/((R'_2)^2 + X^2)/\omega_s = 48.9 \text{N} \cdot \text{m}$ (with the stator impedance of 3.1 included).
- With: replacing R'_2 by 2.0Ω in the same expression gives $T_{start} = 176.8 \text{N} \cdot \text{m} - 3.6\times$ more.
- Left in circuit at $s = 0.04$, the external resistance dissipates $(1 - s)/s\times$ its share of rotor power – slip-frequency current through 1.7Ω wastes $3I^2 \times 1.7 \times (1 - s)$ of it as heat.
- So the running machine short-circuits the slip rings or drops the resistance as speed rises: high starting torque AND low running loss.
- The trade is the wound-rotor machine's reason to exist – cage machines achieve a milder version with double-cage rotors.