9.2 DC machines

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS back-EMF

Objective

Analyse DC motors (shunt/series) and generators from the two machine equations, compute speed control ranges, and verify every answer through $T\omega = EI_a$.

1 Key knowledge and worked examples

■ The two equations and the circuit

$E = k\Phi\omega$ (back-EMF), $T = k\Phi I_a$; the armature circuit gives $V = E + I_a R_a$. Shunt machines: field in parallel ($\Phi \approx$ constant); series: field in series ($\Phi \propto I_a$ until saturation).

■ Worked example: the shunt motor, double-checked

220 V, $R_a = 0.2\Omega$, $I_a = 50A$ at 1500 rpm: $E = 210V$; $k\Phi = E/\omega = 210/157.1 = 1.337$; $T = 66.85N \cdot m$. The check: $T\omega = 66.85 \times 157.1 = 10500W = E \cdot I_a = 210 \times 50$ ✓ – the air gap converts exactly between the electrical and mechanical statements.

Speed control: field weakening raises ω (E = k$\Phi\omega$ at constant E); armature voltage control lowers ω proportionally. Halving V halves E (at the same I_a) $\rightarrow$ ω halves; halving Φ doubles ω (until saturation of the iron limits the weakening range).

■ The series motor

$\Phi \propto I_a$: $T \propto I_a^2$ (before saturation) – the high-starting-torque machine of traction. Danger: at NO load $I_a \rightarrow 0$, $\Phi \rightarrow 0$, $\omega \rightarrow$ runaway; series motors must never be belt-coupled to a load that can fall off.

2 Practice

2.1 A shunt motor at 440 V, $R_a = 0.4\Omega$, $I_a = 30A$, runs at 1200 rpm. Compute E and kΦ (in V $\cdot$ s). [3]

2.2 For 2.1, compute the torque and verify via $T\omega = EI_a$. [4]

2.3 The motor of 2.1 has its armature voltage halved to 220 V (same load torque – state the assumption about I_a). Compute the new E and speed. [3]

3 Exam-style questions

3.1 A 230 V shunt motor: $R_a = 0.25\Omega$, field adjusted so $k\Phi = 1.8V \cdot s$; it drives a constant-torque load of 90 N $\cdot$ m.

Compute I_a (from T), E, and the speed in rpm. Then compute the starting current with full voltage (E = 0) and the ratio to the running current; compute the voltage that limits the starting current to twice running. The load torque stays 90 N $\cdot$ m while the field weakens 20%: compute the new I_a and new speed, and state what limits the weakening in practice. [8]

3.2 A series motor runs at 500 rpm from 230 V, drawing 40 A; total series resistance (armature + field) 0.5 Ω. (From these, $k\Phi$ follows.)

Compute E, derive $k\Phi$ from the operating point, and compute the torque; verify $T\omega = EI_a$. The load drops so the current falls to 20 A, where the magnetisation curve gives $k\Phi = 2.4V \cdot s$ (the field has weakened – the machine is less saturated). Compute the new E and the new speed; explain the speed rise physically. State why an unloaded series motor is dangerous and the standard coupling rule. [7]

Solutions

2.1

- $E = 440 - 30 \times 0.4 = 428V$.
- $\omega = 1200 \times 2\pi/60 = 125.7 rad/s$; $k\Phi = 428/125.7 = 3.405 V \cdot s$.

2.2

- $T = 3.405 \times 30 = 102.1 N \cdot m$.
- $T\omega = 102.1 \times 125.7 = 12838 W$.
- $EI_a = 428 \times 30 = 12840 W$ ✓ (rounding).

2.3

- Same load torque → same T → same $k\Phi \cdot I_a$ → same $I_a = 30A$ (Φ unchanged by armature voltage in a shunt motor).
- New $E = 220 - 30 \times 0.4 = 208V$.
- New speed: $\omega = 208/3.405 = 61.1 rad/s$ → `584 rpm` – just under half.

3.1

- $I_a = T/k\Phi = 90/1.8 = 50A$.
- $E = 230 - 50 \times 0.25 = 217.5V$; $\omega = 217.5/1.8 = 120.8 rad/s$ → `1153 rpm`.
- Starting: $I_{st} = 230/0.25 = 920A$ – 18.4× running.
- Limit to 100 A: $V = 100 \times 0.25 = 25V$ (a soft-start or resistor bank).
- Field −20%: $k\Phi = 1.44$; $I_a = 90/1.44 = 62.5A$ (torque demands more current); $E = 230 - 62.5 \times 0.25 = 214.4V$; $\omega = 214.4/1.44 = 148.9 rad/s$ → 1421 rpm (+23%).
- Practical limit: commutation (sparking as field weakens) and armature reaction; also the rising I_a heats the armature – manufacturers state a maximum weakening (e.g. to 60–80% of full field).

3.2

- $E = 230 - 40 \times 0.5 = 210V$.
- $\omega = 500 rpm = 52.36 rad/s$; $k\Phi = E/\omega = 210/52.36 = 4.01 V \cdot s$; $T = 4.01 \times 40 = 160.4 N \cdot m$.
- Converted power: $EI_a = 210 \times 40 = 8400 W$; `Tω = 160.4 × 52.36 = 8398 W` ✓.
- At 20 A: $E = 230 - 20 \times 0.5 = 220V$; with $k\Phi = 2.4 V \cdot s$: $\omega = 220/2.4 = 91.7 rad/s$ → `876 rpm` – the speed rises ~75%.
- Physically: less current → less flux → the machine must spin faster to develop the same E against the supply.
- Unloaded: $I_a \to 0 \Rightarrow \Phi \to 0 \Rightarrow$ runaway speed destroys the armature (centrifugal forces); the rule – series motors couple only through GEARS or direct shafts, never belts (a slipping belt IS no load).