

9.1 Magnetic circuits and transformers

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS reluctance • core loss

Objective

Solve magnetic circuits, analyse ideal and real transformers, and separate losses by test – every answer checked by the power balance.

1 Key knowledge and worked examples

■ The magnetic Ohm's law

$\Phi = F/R_m$ with $F = NI$ (ampere-turns) and $R_m = l/(\mu A)$ (per henry⁻¹). Flux density $B = \Phi/A$; saturation caps B near ~1.5–1.8 T in silicon steel.

■ Worked example: the core, verified in range

200 turns, 0.5 A, core $l = 0.5$ m, $A = 10$ cm², $\mu_r = 4000$: $R_m = 0.5/(4000\mu_0 \times 0.001) = 99472 \text{ At/Wb}$; $\Phi = 100/99472 = 1.005 \text{ mWb}$; $B = 1.005 \text{ T}$ ✓ – the range check (well under saturation) is part of the answer. Halving the air-gap-free path or doubling μ halves R_m and doubles flux – the design lever.

■ The transformer and its two tests

Ideal: $V_1/V_2 = N_1/N_2$, $I_1/I_2 = N_2/N_1$, lossless. Real: the **OC test** (open-circuit, rated voltage) measures **core loss** (constant with load); the **SC test** (short-circuit, reduced voltage, rated current) measures full-load copper loss.

Worked example – 10 kVA, 2000/400 V: OC 400 V / 0.7 A / 240 W; SC 80 V / 5 A / 300 W. At full load: $\eta = 10000/(10000 + 240 + 300) = 94.9\%$. Balance check: output + both losses = input ✓. At HALF load the core loss stays 240 W but copper drops to a quarter (75 W): $\eta = 5000/(5000 + 315) = 94.1\%$ – the efficiency curve is flat because the losses are split between constant and quadratic terms.

2 Practice

2.1 A core with $R_m = 50000 \text{ At/Wb}$ carries 300 turns at 0.4 A: compute Φ ; with $A = 8 \text{ cm}^2$, compute B and state whether it is near saturation. [3]

2.2 A 2000/400 V transformer feeds a 5Ω load on the secondary. Compute the secondary current and the primary current (ideal). [3]

2.3 A transformer's OC test reads 480 W and SC test 620 W. Compute the full-load efficiency at 25 kVA, 0.8 power factor. [3]

3 Exam-style questions

3.1 A single-phase 50 Hz transformer, 25 kVA, 2200/220 V, gave: OC test (220 V, 4 A, 260 W); SC test (95 V, rated current, 510 W).

Compute the rated currents both sides. Determine the core loss and the full-load copper loss from the tests (state which test gives which). Compute the full-load efficiency at unity pf and at 0.8 pf – explain why one is lower using the loss structure. Compute the all-day efficiency if the transformer is at full load 8 h, half load 8 h, and energised-but-no-load 8 h. [8]

3.2 A 100 kVA transformer has core loss 500 W and full-load copper loss 1200 W.

Derive the load fraction (in kVA, at unity pf) at which efficiency is MAXIMUM (where variable loss = constant loss), and compute that maximum efficiency. Verify by computing the efficiency at load fractions 0.5 and 0.9 – both should be lower. State why the maximum condition equates the two loss families. [7]

Solutions

2.1

- $\Phi = 300 \times 0.4 / 50000 = 2.4 \text{ mWb}$.
- $B = 2.4 \times 10^{-3} / (8 \times 10^{-4}) = 3.0 \text{ T}$ – far PAST typical saturation ($\sim 1.7 \text{ T}$): the core saturates, μ_r collapses, and the linear computation is invalid.

2.2

- Secondary: $400/5 = 80 \text{ A}$.
- Primary: $80 \times 400 / 2000 = 16 \text{ A}$ (or power: $32 \text{ kVA} / 2200 \text{ V} \approx 14.5 \text{ A}$... careful: $400 \times 80 = 32 \text{ kVA}$ exceeds the presumed rating but the ideal relation holds: $I_1 = I_2 N_2 / N_1 = 16 \text{ A}$).

2.3

- Output at 25 kVA , $0.8 \text{ pf} = 20\,000 \text{ W}$.
- $\eta = 20000 / (20000 + 480 + 620) = 94.8\%$.

3.1

- Rated: HV $25\,000 / 2200 = 11.36 \text{ A}$; LV $25\,000 / 220 = 113.6 \text{ A}$.
- Core loss = OC test = 260 W (measured at rated voltage).
- Copper at full load = SC test = 510 W (measured at rated current).
- Full load, unity pf: $\eta = 25000 / (25000 + 770) = 97.0\%$.
- At 0.8 pf the OUTPUT is $20\,000 \text{ W}$ with the SAME losses: $\eta = 20000 / 20770 = 96.3\%$ – lower because the fixed losses now ride on less real power.
- All-day: energies – 8h full: out 200 kWh , loss $0.77 \times 8 = 6.16 \text{ kWh}$; 8h half: out 100 kWh , loss $0.26 + 0.510 \times 0.25 = 0.3875 \text{ kW} \times 8 = 3.1 \text{ kWh}$; 8h no-load: out 0 , loss $0.26 \times 8 = 2.08 \text{ kWh}$. $\eta_{\text{day}} = 300 / (300 + 11.34) = 96.4\%$.

3.2

- Condition: copper loss ($= 1200 \times \text{load}^2$) equals core loss (500): $1200x^2 = 500 \rightarrow x = \sqrt{500/1200} = 0.6455 \rightarrow 64.55 \text{ kVA}$.
- At that point: output 64.55 kW , total loss $500 + 500 = 1000 \text{ W}$: $\eta_{\text{max}} = 64550 / 65550 = 98.47\%$.
- At 0.5 : loss $500 + 1200 \times 0.25 = 800$; $\eta = 50000 / 50800 = 98.43\%$ ✓ lower.
- At 0.9 : loss $500 + 1200 \times 0.81 = 1472$; $\eta = 90000 / 91472 = 98.39\%$ ✓ lower.
- Why equate: efficiency $\eta = xS / (xS + F + cx^2)$ is maximised where its derivative vanishes – d/dx gives $F = cx^2$, the condition that constant loss equals variable loss (the same optimum structure as loading a source at matched loss).