7.7 Waveguides and antennas

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS cutoff • waveguide wavelength • Friis equation

Objective

Compute waveguide cutoff and guide wavelength, and apply the Friis link budget – every guide answer checked against the free-space limit and every link budget against the isotropic baseline.

1 Key knowledge and worked examples

■ Cutoff: the pipe's admission price

A rectangular guide (wide dimension a) supports the dominant TE_{10} mode only above $f_c = c/(2a)$. Below cutoff the wave evanesces – it decays, not propagates. The **guide wavelength** stretches beyond free space:

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

■ Worked example: WR-90 at 10 GHz

$a = 22.86mm \rightarrow f_c = 3\times10^8/(2\times0.02286) = 6.56GHz$. At 10 GHz ($\lambda = 30mm$): $\lambda_g = 30/\sqrt{1-0.656^2} = 39.8mm$ – LONGER than free space because the wave zigzags down the guide (the component along the axis is what advances). Checks: as $f \rightarrow \infty$, $\lambda_g \rightarrow \lambda$ (free space) ✓; as $f \rightarrow f_c^+$, $\lambda_g \rightarrow \infty$ (the wave stops advancing) ✓ – the two limits bracket every guide answer.

■ Antennas and the Friis budget

The half-wave dipole: gain 2.15 dBi, input resistance ≈ 73 Ω. **Friis** $P_r/P_t = G_rG_t(\lambda/4\pi R)^2$ – the inverse-square law with antenna gains. **Worked example**: two 10 dBi (G = 10) antennas 1 km apart at 3 GHz ($\lambda = 0.1m$): $10\times10\times(0.1/(4\pi\times1000))^2 = 6.33\times10^{-9}$ – an 82 dB path loss. Check the isotropic baseline: without gains, `(0.1/12566)^2 = 6.33×10^-11` – 20 dB less (exactly the two 10-dBi gains) ✓.

2 Practice

2.1 A guide with $a = 47.55mm$ (WR-187): compute the cutoff frequency of the dominant mode. [2]

2.2 WR-90 ($f_c = 6.56GHz$) at 12 GHz ($\lambda = 25mm$): compute λ_g. [3]

2.3 Two isotropic antennas 100 m apart at 1.5 GHz: compute the path loss in dB. [3]

3 Exam-style questions

3.1 A rectangular guide $a = 15cm$ is filled with a dielectric ($\varepsilon_r = 2.1$, $\mu_r = 1$).

Compute the cutoff frequency of the TE_{10} mode (remember $v = c/\sqrt{\varepsilon_r}$ in the dielectric), the guide wavelength at 1.5 GHz, and the wave's phase velocity $v_p = f\cdot\lambda_g$ – explain why v_p exceeds the dielectric's light speed and state what the group (energy) velocity does instead. Verify both limits ($f \rightarrow f_c^+$ and $f \gg f_c$). [7]

3.2 A satellite link: 12 GHz downlink, satellite EIRP 50 dBW (that is, $P_tG_t = 10^5 W$ effective), receive dish gain 45 dBi, range 38,000 km.

Compute the free-space path loss at 12 GHz ($\lambda = 25$ mm) in dB, the received power in dBW, and in watts. Then compute the noise floor for a 20 MHz bandwidth receiver at 100 K system temperature (`N = kTB`, k = 1.38×10^{-23}), and the link margin in dB. State whether the link closes with a 3 dB requirement for rain fade. [7]

Solutions

2.1

- $f_c = c/(2a) = 3 \times 10^8/(2 \times 0.04755) = 3.15GHz$.

2.2

- $\lambda_g = 25/\sqrt{(1 - (6.56/12)^2)} = 25/\sqrt{1 - 0.299} = 25/0.838 = 29.8mm$.

2.3

- $\lambda = 3 \times 10^8/1.5 \times 10^9 = 0.2m$; $(\lambda/4\pi R)^2 = (0.2/(4\pi \times 100))^2 = 2.53 \times 10^{-8}$.
- Path loss $= -10log_{10}(2.53 \times 10^{-8}) = 76.0dB$.

3.1

- In the dielectric $v = c/\sqrt{2.1} = 2.07 \times 10^8$ m/s: $f_c = v/(2a) = 2.07 \times 10^8/0.30 = 0.69$ GHz.
- At 1.5 GHz: $\lambda = v/f = 0.138m$; $\lambda_g = 0.138/\sqrt{(1 - (0.69/1.5)^2)} = 0.138/\sqrt{0.788} = 0.155m$.
- $v_p = f \cdot \lambda_g = 1.5 \times 10^9 \times 0.155 = 2.33 \times 10^8 m/s > v = 2.07 \times 10^8$ – the phase fronts sweep along the axis faster than light-in-the-medium travels; the ENERGY moves at the group velocity $v_g = v^2/v_p = 1.84 \times 10^8 m/s$ (slower than v, the zigzag penalty).
- Limits: $f \to f_c^+$: $\lambda_g \to \infty$, $v_p \to \infty$, $v_g \to 0$ (energy stops) ✓; $f \gg f_c$: $\lambda_g \to \lambda$, both velocities $\to$ v (free space) ✓.

3.2

- Path loss: $(\lambda/4\pi R)^2 = (0.025/(4\pi \times 3.8 \times 10^7))^2 = 2.74 \times 10^{-21}$ – $-10\log_{10}(2.74 \times 10^{-21}) = 205.6$ dB.
- Received power: `50 dBW - 205.6 + 45 = -110.6 dBW`.
- In watts: $10^{-110.6/10} = 8.71 \times 10^{-12}$ W (8.7 pW).
- Noise: $N = 1.38 \times 10^{-23} \times 100 \times 20 \times 10^6 = 2.76 \times 10^{-14}W = -135.6dBW$.
- Margin: `-110.6 - (-135.6) = 25.0 dB` – the link closes with 25 dB to spare, far above the 3 dB rain-fade requirement ✓.