

7.7 Waveguides and antennas

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS cutoff • waveguide wavelength • Friis equation

Objective

Compute waveguide cutoff and guide wavelength, and apply the Friis link budget – every guide answer checked against the free-space limit and every link budget against the isotropic baseline.

1 Key knowledge and worked examples

■ Cutoff: the pipe's admission price

A rectangular guide (wide dimension a) supports the dominant TE_{10} mode only above $f_c = c/(2a)$. Below cutoff the wave evanesces – it decays, not propagates. The **guide wavelength** stretches beyond free space:

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$$

■ Worked example: WR-90 at 10 GHz

$a = 22.86\text{mm} \rightarrow f_c = 3 \times 10^8 / (2 \times 0.02286) = 6.56\text{GHz}$. At 10 GHz ($\lambda = 30\text{mm}$): $\lambda_g = 30 / \sqrt{1 - 0.656^2} = 39.8\text{mm}$ – LONGER than free space because the wave zigzags down the guide (the component along the axis is what advances). Checks: as $f \rightarrow \infty$, $\lambda_g \rightarrow \lambda$ (free space) ✓; as $f \rightarrow f_c^+$, $\lambda_g \rightarrow \infty$ (the wave stops advancing) ✓ – the two limits bracket every guide answer.

■ Antennas and the Friis budget

The half-wave dipole: gain 2.15 dBi, input resistance $\approx 73 \Omega$. **Friis**

$P_r/P_t = G_r G_t (\lambda/4\pi R)^2$ – the inverse-square law with antenna gains. **Worked example:** two 10 dBi ($G = 10$) antennas 1 km apart at 3 GHz ($\lambda = 0.1\text{m}$): $10 \times 10 \times (0.1/(4\pi \times 1000))^2 = 6.33 \times 10^{-9}$ – an 82 dB path loss. Check the isotropic baseline: without gains, $(0.1/12566)^2 = 6.33 \times 10^{-11}$ – 20 dB less (exactly the two 10-dBi gains) ✓.

2 Practice

2.1 A guide with $a = 47.55\text{mm}$ (WR-187): compute the cutoff frequency of the dominant mode. [2]

2.2 WR-90 ($f_c = 6.56\text{GHz}$) at 12 GHz ($\lambda = 25\text{mm}$): compute λ_g . [3]

2.3 Two isotropic antennas 100 m apart at 1.5 GHz: compute the path loss in dB. [3]

3 Exam-style questions

3.1 A rectangular guide $a = 15\text{cm}$ is filled with a dielectric ($\epsilon_r = 2.1$, $\mu_r = 1$).

Compute the cutoff frequency of the TE_{10} mode (remember $v = c/\sqrt{\epsilon_r}$ in the dielectric), the guide wavelength at 1.5 GHz, and the wave's phase velocity $v_p = f \cdot \lambda_g$ – explain why v_p exceeds the dielectric's light speed and state what the group (energy) velocity does instead. Verify both limits ($f \rightarrow f_c^+$ and $f \gg f_c$). [7]

3.2 A satellite link: 12 GHz downlink, satellite EIRP 50 dBW (that is, $P_t G_t = 10^5 W$ effective), receive dish gain 45 dBi, range 38,000 km.

Compute the free-space path loss at 12 GHz ($\lambda = 25$ mm) in dB, the received power in dBW, and in watts. Then compute the noise floor for a 20 MHz bandwidth receiver at 100 K system temperature ($N = kTB$, $k = 1.38 \times 10^{-23}$), and the link margin in dB. State whether the link closes with a 3 dB requirement for rain fade. [7]

Solutions

2.1

- $f_c = c/(2a) = 3 \times 10^8 / (2 \times 0.04755) = 3.15 \text{ GHz}$.

2.2

- $\lambda_g = 25 / \sqrt{1 - (6.56/12)^2} = 25 / \sqrt{1 - 0.299} = 25 / 0.838 = 29.8 \text{ mm}$.

2.3

- $\lambda = 3 \times 10^8 / 1.5 \times 10^9 = 0.2 \text{ m}$; $(\lambda/4\pi R)^2 = (0.2/(4\pi \times 100))^2 = 2.53 \times 10^{-8}$.
- Path loss = $-10 \log_{10}(2.53 \times 10^{-8}) = 76.0 \text{ dB}$.

3.1

- In the dielectric $v = c/\sqrt{2.1} = 2.07 \times 10^8 \text{ m/s}$: $f_c = v/(2a) = 2.07 \times 10^8 / 0.30 = 0.69 \text{ GHz}$.
- At 1.5 GHz: $\lambda = v/f = 0.138 \text{ m}$; $\lambda_g = 0.138 / \sqrt{1 - (0.69/1.5)^2} = 0.138 / \sqrt{0.788} = 0.155 \text{ m}$.
- $v_p = f \cdot \lambda_g = 1.5 \times 10^9 \times 0.155 = 2.33 \times 10^8 \text{ m/s} > v = 2.07 \times 10^8$ – the phase fronts sweep along the axis faster than light-in-the-medium travels; the ENERGY moves at the group velocity $v_g = v^2/v_p = 1.84 \times 10^8 \text{ m/s}$ (slower than v , the zigzag penalty).
- Limits: $f \rightarrow f_c^+$: $\lambda_g \rightarrow \infty$, $v_p \rightarrow \infty$, $v_g \rightarrow 0$ (energy stops) ✓; $f \gg f_c$: $\lambda_g \rightarrow \lambda$, both velocities $\rightarrow v$ (free space) ✓.

3.2

- Path loss: $(\lambda/4\pi R)^2 = (0.025/(4\pi \times 3.8 \times 10^7))^2 = 2.74 \times 10^{-21}$ – $-10 \log_{10}(2.74 \times 10^{-21}) = 205.6 \text{ dB}$.
- Received power: $50 \text{ dBW} - 205.6 + 45 = -110.6 \text{ dBW}$.
- In watts: $10^{-110.6/10} = 8.71 \times 10^{-12} \text{ W}$ (8.7 pW).
- Noise: $N = 1.38 \times 10^{-23} \times 100 \times 20 \times 10^6 = 2.76 \times 10^{-14} \text{ W} = -135.6 \text{ dBW}$.
- Margin: $-110.6 - (-135.6) = 25.0 \text{ dB}$ – the link closes with 25 dB to spare, far above the 3 dB rain-fade requirement ✓.