UNIVERSITY ELECTRICAL ENGINEERING • EXERCISE SHEET

7.6 Transmission lines

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS characteristic impedance • bounce diagram

Objective

Compute Z_0 and wave speed from line parameters, track steps with the bounce diagram, match loads, and read the Smith chart's basic moves – every bounce answer checked by VSWR or energy conservation.

1 Key knowledge and worked examples

■ **The telegrapher pair**

$\partial v/\partial x = -L'\partial i/\partial t$ and $\partial i/\partial x = -C'\partial v/\partial t$ give waves with $v_p = 1/\sqrt{L'C'}$ and **characteristic impedance** $Z_0 = \sqrt{L'/C'}$: the ratio every travelling wave maintains, whatever the load.

■ **Worked example: the 50 Ω coax**

`L' = 250 nH/m`, `C' = 100 pF/m`: $Z_0 = \sqrt{250n/100p} = 50\Omega$; $v_p = 1/\sqrt{250n \times 100p} = 2 \times 10^8 m/s$ (2/3 c – the polyethylene dielectric). A 12 V step launched in: $I = 12/50 = 0.24A$. Open-circuited: the reflection $\Gamma = +1$ doubles the voltage (24 V at the load), cancels the current – and the wave returns to the source to begin the next bounce. A shorted line ($\Gamma = -1$) cancels the voltage and doubles the current. **VSWR** $= (1 + |\Gamma|)/(1 - |\Gamma|)$: 1 matched, ∞ at open/short.

■ **Matching: the quarter-wave trick**

A load Z_L mismatches a line Z_0; a quarter-wave section of impedance $Z_Q = \sqrt{Z_0 Z_L}$ transforms the load into a match (rotated a half-turn on the Smith chart). **Worked example**: matching a 100 Ω antenna to 50 Ω line at 1 GHz – $Z_Q = \sqrt{5000} = 70.7\Omega$, length $\lambda/4 = v_p/4f$ (in the section's dielectric). The Smith chart is these two operations drawn: constant-$|\Gamma|$ circles and the toward-generator rotation.

2 Practice

2.1 A line has `L' = 400 nH/m`, `C' = 40 pF/m`: compute Z_0 and v_p (as a fraction of c). [3]

2.2 A 10 V source with 50 Ω internal resistance drives a 50 Ω line, open-ended: give the launched wave's V and I, and the voltage at the load after one round trip (ignore further source reflections). [3]

2.3 Compute Γ and VSWR for $Z_L = 25\Omega$ on a 50 Ω line. [3]

3 Exam-style questions

3.1 A 75 Ω TV line, 30 m long, with $v_p = 2 \times 10^8 m/s$, feeds a 300 Ω antenna; the source is 75 Ω resistive, launching a 15 V step.

Compute Γ at the load, the launched wave, the first reflected wave (V and I), the voltage at the antenna after the first arrival, and the wave returning to the source (compute the source-side Γ and the second reflection toward the load). Give the one-way transit time. State the steady state the bounces converge to. [8]

3.2 Match a 112.5 Ω load to a 50 Ω line at 900 MHz using a quarter-wave section with $v_p = 2 \times 10^8 m/s$.

Compute the section's required impedance and physical length. Verify the match by computing the input impedance of the section+load ($Z_{in} = Z_Q^2/Z_L$) and the resulting Γ at the junction. Then state the bandwidth behaviour (what happens as frequency moves away from the design point) and why the match is exact only at the design frequency. [7]

Solutions

2.1

- $Z_0 = \sqrt{400n/40p} = \sqrt{10000} = 100\Omega$.
- $v_p = 1/\sqrt{400n \times 40p} = 1/\sqrt{1.6 \times 10^{-17}} = 2.5 \times 10^8 m/s = 0.83c$.

2.2

- Divider at launch: $V = 10 \times 50/(50 + 50) = 5V$, $I = 5/50 = 0.1A$.
- Open load: $\Gamma_L = +1$ – voltage doubles to 10 V at the load.
- Back at the source (matched, $\Gamma_S = 0$): no re-reflection; the line settles with the source-end wave absorbed.

2.3

- $\Gamma = (25 - 50)/(25 + 50) = -1/3$.
- $VSWR = (1 + 1/3)/(1 - 1/3) = 2$.

3.1

- $\Gamma_L = (300 - 75)/(300 + 75) = 0.6$.
- Launched: $V = 15 \times 75/(75 + 75) = 7.5V$, $I = 0.1A$.
- Reflected: $V_r = 0.6 \times 7.5 = 4.5V$; $I_r = -0.6 \times 0.1 = -0.06A$ (reflected current carries the opposite sign).
- Load voltage after first arrival: `7.5 + 4.5 = 12 V`.
- Source-side Γ: $\Gamma_S = (75 - 75)/(75 + 75) = 0$ – the returning wave is absorbed; NO second reflection.
- Transit time: $30/(2 \times 10^8) = 150ns$ one way.
- Steady state: with the source matched, the bounces end after one round trip; the load voltage settles at 12 V (and current 0.04 A – the divider $15 \times 300/(75{+}300) = 12$ V ✓ the DC answer).

3.2

- $Z_Q = \sqrt{50 \times 112.5} = \sqrt{5625} = 75\Omega$.
- Length: $\lambda/4 = v_p/4f = 2 \times 10^8/(4 \times 9 \times 10^8) = 5.56cm$.
- $Z_{in} = Z_Q^2/Z_L = 75^2/112.5 = 5625/112.5 = 50\Omega$ – exactly matched ✓.
- Γ at the junction: `(50 - 50)/(50 + 50) = 0` ✓.
- Bandwidth: away from the design frequency the section is no longer a quarter wavelength, so the rotation angle is wrong and the transformed impedance slides off 50 Ω – $|\Gamma|$ grows smoothly; the match is exact only where the section is exactly $\lambda/4$.