7.5 Plane waves

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS plane wave • skin depth

Objective

Relate E, B, wavelength, speed and power for plane waves; compute reflection at normal incidence; and quantify skin depth in conductors – every answer checked against free-space values and material limits.

1 Key knowledge and worked examples

■ The plane wave trio

In lossless space: `E` ⊥ `B` ⊥ `k`, `B = E/v`, $v = 1/\sqrt{\mu\varepsilon} = c$ in vacuum, and the **intrinsic impedance** $\eta = \sqrt{\mu/\varepsilon} = 377\Omega$ links field to power: $S = E^2/\eta$ (the Poynting magnitude).

■ Worked example: one wave, four numbers

A 100 V/m wave at 1 MHz in air: $B = E/c = 0.333\mu T$; $\lambda = c/f = 300m$; $S = 100^2/377 = 26.5W/m^2$. Check the trio: `S = EB/`$\mu_0$ `= 100 × 0.333µ/(4π×10`$^{-7}$`) = 26.5` ✓ – two formulas, one power density.

■ Reflection and the conductor's skin

At a dielectric boundary (normal incidence), the reflection coefficient $\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$. A wave in air hitting a perfect conductor ($\eta_2 = 0$): $\Gamma = -1$ – total reflection with a sign flip, E doubles at the surface (standing wave), B cancels. In a **good conductor** the field decays over the **skin depth** $\delta = \sqrt{(2/(\omega\mu\sigma))}$: copper at 50 Hz gives $\delta = 9.3$ mm, at 1 GHz $\delta = 2.1$ µm – the numbers that explain shields, RF surface current and litz wire.

2 Practice

2.1 A 50 V/m wave at 100 MHz in air: compute B, λ and S. [3]

2.2 A wave passes from air into a lossless dielectric with $\varepsilon_r = 4$ ($\mu_r = 1$): compute the new speed and wavelength, and the reflection coefficient at the boundary. [4]

2.3 Compute the skin depth in copper ($\sigma = 5.8\times10^7$ S/m) at 10 MHz. [2]

3 Exam-style questions

3.1 A 200 V/m, 300 MHz plane wave in air hits a lossless medium with $\varepsilon_r = 9, \mu_r = 1$ at normal incidence.

Compute η in both media, the reflection and transmission coefficients (Γ and $\tau = 1 + \Gamma$ for E), the reflected and transmitted E magnitudes, and the standing-wave ratio in air. Verify the power split: reflected fraction Γ^2 plus transmitted fraction must equal 1 (use the transmitted power formula $S_t/S_i = (\eta_1/\eta_2)\tau^2$). [7]

3.2 An aluminium shield ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$) of thickness 1 mm must attenuate a 1 kHz magnetic field by at least a factor of 100.

Compute the skin depth at 1 kHz, the attenuation $e^{-t/\delta}$ of one pass through 1 mm, and the number of skin depths the thickness represents. State whether one pass suffices for the 100× requirement; if not, compute the required thickness. Explain (one sentence) why low-frequency magnetic fields are the hardest to shield. [7]

Solutions

2.1

- $B = 50/(3 \times 10^8) = 0.167 \mu T$.
- $\lambda = 3 \times 10^8 / 10^8 = 3m$.
- $S = 50^2/377 = 6.63 W/m^2$.

2.2

- $v = c/\sqrt{\varepsilon_r} = c/2 = 1.5 \times 10^8 m/s$; $\lambda = \lambda_0/2$ (frequency unchanged).
- $\eta_2 = \eta_0/2 = 188.5\Omega$; $\Gamma = (188.5 - 377)/(188.5 + 377) = -1/3$.

2.3

- $\delta = \sqrt(2/(2\pi \times 10^7 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7)) = \sqrt(2/(4.58 \times 10^9)) = 20.9$ μm.

3.1

- $\eta_1 = 377\Omega$; $\eta_2 = 377/\sqrt{9} = 125.7\Omega$.
- $\Gamma = (125.7 - 377)/(125.7 + 377) = -0.5$; $\tau = 1 + \Gamma = 0.5$.
- Reflected E: $|\Gamma| \times 200 = 100 V/m$; transmitted: $0.5 \times 200 = 100 V/m$.
- SWR $= (1 + |\Gamma|)/(1 - |\Gamma|) = 1.5/0.5 = 3$.
- Power: reflected $\Gamma^2 = 0.25$; transmitted $S_t/S_i = (\eta_1/\eta_2)\tau^2 = 3 \times 0.25 = 0.75$; `0.25 + 0.75 = 1` ✓ energy conserved.

3.2

- $\delta = \sqrt(2/(2\pi \times 10^3 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7)) = \sqrt(2/(2.76 \times 10^5)) = 2.69 mm$.
- One pass through 1 mm: $e^{-1/2.69} = e^{-0.372} = 0.689$ – only a 31% attenuation.
- 1 mm is `1/2.69 = 0.37` skin depths – far too thin.
- For 100× (factor 0.01): $t = \delta \cdot ln(100) = 2.69 \times 4.605 = 12.4 mm$.
- Low-frequency magnetic fields are hardest to shield because $\delta \propto 1/\sqrt{}$f: the skin depth grows as frequency falls, so practical thicknesses stop working (the reason mu-metal and closed magnetic return paths are used instead of simple conductors).