

7.3 Magnetostatics

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS Biot–Savart law – • Ampère’s law
• magnetic flux • inductance • Lorentz force

Objective

Compute B fields by Biot–Savart and Ampère, magnetic flux, inductance, and Lorentz forces – every Ampère answer cross-checked where possible by direct integration or a limiting case.

1 Key knowledge and worked examples

■ Two routes to B

Biot–Savart – (always works, usually hard): $dB = \mu_0 Idl \times \hat{r}/(4\pi r^2)$. **Ampère’s law** (when symmetry makes B constant on a loop): $\oint B \cdot dl = \mu_0 I_{enc}$.

■ Worked example: the infinite solenoid, and its partner

Inside: $B = \mu_0 nI$ uniform; outside ≈ 0 . With $n = 1000$ turns/m, $I = 2A$: $B = 4\pi \times 10^{-7} \times 2000 = 2.51mT$. The partner – the infinite straight wire: $B = \mu_0 I/(2\pi r)$: at 5 cm from a 10 A wire, $B = 4\pi \times 10^{-7} \times 10/(2\pi \times 0.05) = 40\mu T$ (the Earth-field scale). Both are Ampère answers; Biot–Savart on the solenoid would be a page of integrals – the symmetry test comes first, always.

■ Flux, inductance, force

Magnetic flux $\Phi = \int B \cdot dA$; **inductance** $L = N\Phi/I$ (the solenoid: $L/l = \mu_0 n^2 A$). The **Lorentz force** $F = qv \times B$ on charges, $F = Il \times B$ on wires – magnitude $qvB\sin\theta$, direction by the right hand. A 1 m wire carrying 10 A perpendicular to 0.5 T feels $10 \times 1 \times 0.5 = 5N$ – motor forces are ampere-scale currents times laboratory fields.

2 Practice

2.1 B at 20 cm from a long straight wire carrying 15 A. [2]

2.2 A solenoid has 500 turns over 10 cm, radius 2 cm, $I = 3A$: compute B inside and the total flux (assume uniform inside, zero outside). [3]

2.3 A 2 m wire carrying 8 A lies perpendicular to a 0.3 T field: force magnitude and direction rule. [2]

3 Exam-style questions

3.1 A toroid: $N = 200$ turns, inner radius 5 cm, outer radius 7 cm, $I = 1.5A$.

Using Ampère’s law on a circular path of radius r inside the core, derive B(r) for $5 < r < 7$ cm, evaluate at both edges, and explain why B is NOT uniform in the core (contrast the solenoid). Then compute the flux through a cross-section by integrating B(r) over the area (core depth d = 2 cm, rectangular section), and estimate the inductance $L = N\Phi/I$ (leave the answer in μH with the integral evaluated numerically by the midpoint rule at r = 6 cm as a check). [8]

3.2 A rectangular loop ($10\text{ cm} \times 15\text{ cm}$) carries 2 A ; its long side lies parallel to and 4 cm from a long wire carrying 12 A .

Compute the force on the NEAR long side and on the FAR long side (at 14 cm), state their directions, and find the net force on the loop. Then compute the flux through the loop from the wire's field (integrate $\mathbf{B}(\mathbf{r}) \cdot d\mathbf{r}$ over the $4\text{--}14\text{ cm}$ span, loop length 15 cm) and the mutual inductance $M = \Phi/I_{\text{wire}}$. [7]

Solutions

2.1

- $B = \mu_0 I / (2\pi r) = 4\pi \times 10^{-7} \times 15 / (2\pi \times 0.2) = 15\mu T$.

2.2

- $n = 500/0.10 = 5000/m$; $B = \mu_0 n I = 4\pi \times 10^{-7} \times 15000 = 18.8mT$.
- $\Phi = B \cdot A = 18.8m \times \pi(0.02)^2 = 23.7\mu Wb$.

2.3

- $F = BIl = 0.3 \times 8 \times 2 = 4.8N$.
- Direction: $Il \times B$ – right-hand rule from current to field.

3.1

- Ampère: $B \cdot 2\pi r = \mu_0 NI \rightarrow B(r) = \mu_0 NI / (2\pi r)$.
- At $r = 5$ cm: $4\pi \times 10^{-7} \times 300 / (2\pi \times 0.05) = 1.20mT$; at $r = 7$ cm: 0.857 mT – B FALLS with r across the core (the outer path is longer for the same enclosed current), unlike the solenoid's uniform interior.
- Flux: $\Phi = \int_{0.05}^{0.07} B(r) \cdot d \cdot dr = (\mu_0 NI d / 2\pi) \ln(7/5)$.
- $= (4\pi \times 10^{-7} \times 200 \times 1.5 \times 0.02 / (2\pi)) \times 0.3365 = 4.04 \times 10^{-7}$ Wb.
- Midpoint check at $r = 6$ cm: $B = 1.00$ $\mu T \cdot m^{-1}$ -scaled: $B = 1.00$ mT over area $0.02 \times 0.02 = 4.0 \times 10^{-7}$ Wb ✓ (log and midpoint agree to 1%).
- $L = N\Phi / I = 200 \times 4.04 \times 10^{-7} / 1.5 = 53.8$ μH .

3.2

- Near side: $F/l = \mu_0 I_1 I_2 / (2\pi r) = 4\pi \times 10^{-7} \times 12 \times 2 / (2\pi \times 0.04) = 120\mu N/m \rightarrow F = 120\mu \times 0.15 = 18\mu N$ ATTRACTED (parallel currents).
- Far side: at 0.14 m: $34.3\mu N/m \times 0.15 = 5.1\mu N$, repelled.
- Net: $18 - 5.1 = 12.9$ μN toward the wire.
- Flux: $\Phi = (\mu_0 I \ell / 2\pi) \ln(14/4) = (4\pi \times 10^{-7} \times 12 \times 0.15 / (2\pi)) \times 1.2528 = 4.52 \times 10^{-7}$ Wb.
- $M = \Phi / I_{wire} = 4.52 \times 10^{-7} / 12 = 37.6nH$.