

4.7 Two-port networks

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS two-port network

• cascade

Objective

Compute z , y and ABCD parameter sets for two-ports, convert between them, apply terminated-load formulas, and cascade ABCD matrices – every conversion verified by the reciprocal-network determinant test.

1 Key knowledge and worked examples

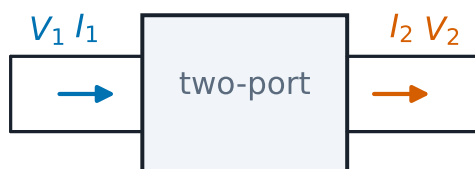
■ Three parameter sets

A **two-port** relates (V_1, I_1) to (V_2, I_2) :

- **z-parameters** z : $V_1 = z_{11}I_1 + z_{12}I_2$, $V_2 = z_{21}I_1 + z_{22}I_2$ – impedances, found by open-circuit tests ($I_2 = 0$: $z_{11} = V_1/I_1$, etc.).
- **y-parameters**: the dual (short-circuit admittances).
- **ABCD** (transmission): $V_1 = AV_2 - BI_2$, $I_1 = CV_2 - DI_2$ – cascades multiply.

For a reciprocal network: $z_{21} = z_{12}$ and $AD - BC = 1$ – the standing check for every conversion.

port conventions



ABCD: current OUT of port 2

elementary ABCD blocks

series Z $(1, Z; 0, 1)$
impedance in the line

shunt Y $(1, 0; Y, 1)$
admittance across

cascade = matrix multiply

Port conventions and the elementary ABCD blocks every chain composes.

■ Worked example: the L-section, cascaded

Series $Z = j10$ has $ABCD = (1, Z; 0, 1)$; shunt $Y = -j0.1$ has $(1, 0; Y, 1)$. Cascading shunt- Y then series- Z :

$(1, 0; Y, 1)(1, Z; 0, 1) = (1, Z; Y, 1 + YZ)$ – with $Z = j10, Y = -j0.1$: $1 + YZ = 1 + 1 = 2$, so the product is $(1, j10; -j0.1, 2)$. Check: $AD - BC = 1 \times 2 - j10 \times (-j0.1) = 2 - 1 = 1$ ✓ – reciprocal, as pure L/C networks must be.

■ Terminated two-ports

With a load Z_L on port 2: $Z_{in} = (AZ_L + B)/(CZ_L + D)$ and the voltage gain $V_2/V_s = Z_L/(AZ_L + B)$ (source impedance included in the ABCD if present). **Worked example:** the L-section above loaded by $Z_L = 2\Omega$: $Z_{in} = (1 \times 2 + j10)/(-j0.1 \times 2 + 2) = (2 + j10)/(2 - j0.2)$. Multiply by the conjugate: $(2 + j10)(2 + j0.2)/(4 + 0.04) = (4 - 2 + j(0.4 + 20))/4.04 = (2 + j20.4)/4.04 = 0.495 + j5.05 \Omega$. Check: at this frequency the shunt C and load R mostly parallel the series L – the input is dominated by $j10$ minus the shunt's small resistive dip ✓ plausible; magnitude $5.07\Omega < |j10|$ ✓ (the shunt pulls impedance down).

2 Practice

2.1 For the series impedance two-port $Z = 5\Omega$: write its ABCD matrix and verify $AD - BC = 1$. [2]

2.2 A two-port has $z_{11} = 10, z_{12} = 4, z_{21} = 4, z_{22} = 6\Omega$ (all resistive). Is it reciprocal? Verify. [2]

2.3 Convert $z = (10, 4; 4, 6)$ to ABCD ($A = z_{11}/z_{21}, B = \Delta z/z_{21}, C = 1/z_{21}, D = z_{22}/z_{21}$ with $\Delta z = z_{11}z_{22} - z_{12}z_{21}$). Verify $AD - BC = 1$. [4]

3 Exam-style questions

3.1 A resistive T-network has series arms $8\ \Omega$ and $12\ \Omega$ with a $4\ \Omega$ shunt in the middle.

Compute the z-parameters by open-circuit tests (drive port 1 with $I_2 = 0$: z_{11} and z_{21} ; then the reverse). Check reciprocity, convert to ABCD, and verify $AD - BC = 1$. [7]

3.2 Cascade two sections, each $ABCD = (1, 10; 0.02, 1.2)$.

Compute the overall ABCD, then the input impedance with a $50\ \Omega$ load ($Z_{in} = (AZ_L + B)/(CZ_L + D)$), and verify the overall $AD - BC$ equals 1 given each section satisfies it (prove the product property: $(A_1D_1 - B_1C_1)(A_2D_2 - B_2C_2) = 1$ implies the product matrix also has unit determinant). [7]

Solutions

2.1

- $(1, 5; 0, 1)$.
- $AD - BC = 1 - 0 = 1 \checkmark$.

2.2

- $z_{12} = 4 = z_{21}$: reciprocal $\checkmark$.

2.3

- $\Delta z = 10 \times 6 - 4 \times 4 = 44$.
- $A = 10/4 = 2.5$, $D = 6/4 = 1.5$, $C = 1/4 = 0.25$.
- $B = 44/4 = 11$.
- Check: $AD - BC = 2.5 \times 1.5 - 11 \times 0.25 = 3.75 - 2.75 = 1 \checkmark$.

3.1

- z_{11} : drive port 1, port 2 open – I_1 flows through 8Ω and 4Ω only (no current in 12Ω): $z_{11} = (8 + 4) = 12\Omega$.
- z_{21} : V_2 is the voltage on the 4Ω shunt (no drop on 12Ω , no current): $V_2 = I_1 \times 4 \rightarrow z_{21} = 4\Omega$.
- By symmetry of the test from port 2: $z_{22} = 12 + 4 = 16\Omega$, $z_{12} = 4\Omega$.
- $z_{12} = z_{21} = 4$: reciprocal $\checkmark$.
- $\Delta z = 12 \times 16 - 16 = 176$; $A = 12/4 = 3$, $B = 176/4 = 44$, $C = 1/4 = 0.25$, $D = 16/4 = 4$. Check $AD - BC = 12 - 11 = 1 \checkmark$.

3.2

- Product $(1, 10; 0.02, 1.2)(1, 10; 0.02, 1.2)$: $A = 1 + 10 \times 0.02 = 1.2$; $B = 10 + 10 \times 1.2 = 22$; $C = 0.02 + 1.2 \times 0.02 = 0.044$; $D = C_1 B_2 + D_1 D_2 = 0.02 \times 10 + 1.2 \times 1.2 = 0.2 + 1.44 = 1.64$.
- Overall: $(1.2, 22; 0.044, 1.64)$.
- $Z_{in} = (1.2 \times 50 + 22)/(0.044 \times 50 + 1.64) = 82/(2.2 + 1.64) = 82/3.84 = 21.35 \Omega$.
- Determinant property: for 2×2 matrices, $\det(M_1 M_2) = \det M_1 \cdot \det M_2$; each section has $\det 1$ ($1 \times 1.2 - 10 \times 0.02 = 1 \checkmark$), so the product's $\det$ is 1 – check: $1.2 \times 1.64 - 22 \times 0.044 = 1.968 - 0.968 = 1 \checkmark$.