

4.5 Filters and Bode plots

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS filter • Bode plot • corner frequency • decibel

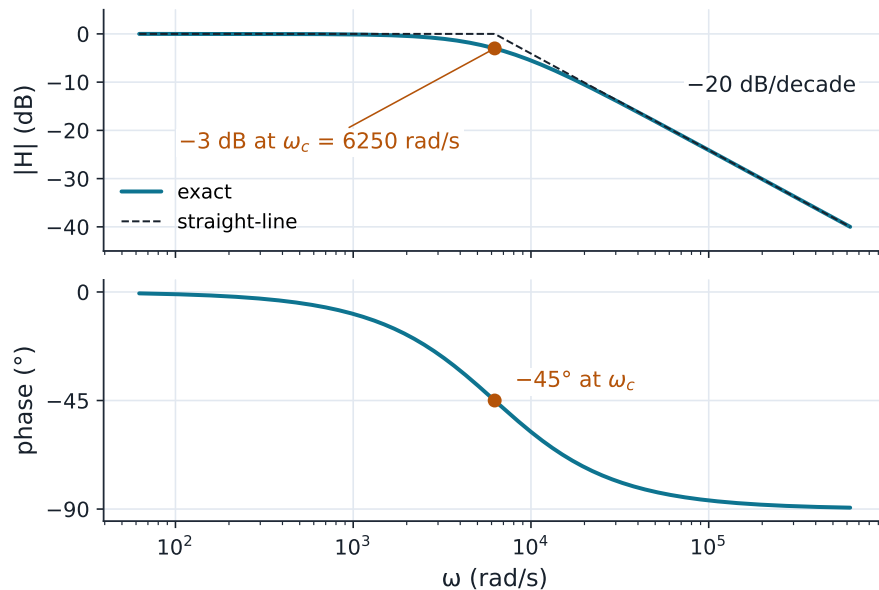
Objective

Design and analyse first/second-order passive filters, sketch straight-line Bode asymptotes with correctly labelled corners, and verify designs against stopband specifications in dB.

1 Key knowledge and worked examples

■ Filter classes and the transfer function

A **filter** transfer function $H(j\omega) = V_{out}/V_{in}$ classifies by its passband: low-pass, high-pass, band-pass (series RLC take-off across R), band-stop (across LC). First-order passive sections roll off at 20 dB/decade; two cascaded (first-order) sections give 40 dB/decade.



The RC low-pass: -3 dB and -45° at the corner, -20 dB per decade after.

■ The Bode plot: straight lines that carry the design

The **Bode plot** plots $20\log_{10}|H|$ (dB) against $\log \omega$. A first-order low-pass RC has magnitude $1/\sqrt{1 + (\omega/\omega_c)^2}$ and the **corner frequency** $\omega_c = 1/(RC)$:

- $\omega \ll \omega_c$: 0 dB (flat); $\omega = \omega_c$: -3 dB, phase -45° ; $\omega \gg \omega_c$: -20 dB/decade, phase $\rightarrow -90^\circ$.

Worked example. $R = 1.6k\Omega$, $C = 0.1\mu F$: $\omega_c = 6250\text{rad/s}$ (995 Hz). At $10\omega_c$ the exact gain is $1/\sqrt{101} = -20.04\text{dB}$ – the straight line reads -20 dB. Each pole -20 dB/decade; each zero +20; dB arithmetic is $20\log_{10}10 = 20$ per decade.

■ Designing against a stopband spec

Worked example. Require at least -40 dB at 10 kHz with an RC low-pass: -40 dB needs $(\omega/\omega_c)^2 \geq 10^4 \rightarrow \omega_c \leq \omega/100$. So a single section needs $\omega_c \leq 2\pi \times 10^4/100 = 628\text{rad/s}$. Check: at $\omega_c = 628$, gain at 10 kHz is $1/\sqrt{1 + 100^2} = 1/100.005 \rightarrow -40.0\text{dB}$ ✓. Two cascaded sections would allow a higher corner (-40 dB/decade), trading simplicity for passband width.

2 Practice

2.1 An RC low-pass has $R = 2\text{ k}\Omega$, $C = 0.05\text{ }\mu\text{F}$. Compute ω_c in rad/s and Hz, and the phase at ω_c . [3]

2.2 For that filter, give the gain in dB at ω_c , at $10\omega_c$ (exact to 1 dp), and at $100\omega_c$ (asymptotic). [3]

2.3 A first-order high-pass RC (output across R) has the same R and C. State its corner and its asymptotic slope at low frequency. [2]

3 Exam-style questions

3.1 Design a first-order RC low-pass that attenuates a 4 kHz tone by at least 30 dB while passing 400 Hz with no more than 1 dB loss.

Derive the corner constraints from BOTH specifications, choose a corner (with $R = 3.3 \text{ k}\Omega$, compute C), and verify the design meets both specs exactly. State the conflict if the two constraints leave no feasible corner. [7]

3.2 A band-pass RLC (series R, output across R) has $R = 50\Omega$, $L = 25mH$, $C = 0.1\mu F$.

Compute ω_0 , Q , the bandwidth, and the two half-power frequencies. Sketch (describe) the Bode magnitude asymptotes: flat 0 dB between the corners, falling -20 dB/decade below the lower and above the upper corner. Verify the peak gain is 0 dB (all source voltage across R at resonance). [7]

Solutions

2.1

- $\omega_c = 1/(2000 \times 5 \times 10^{-8}) = 10000 \text{ rad/s}$.
- $f_c = 10000/2\pi = 1592 \text{ Hz}$.
- Phase at $\omega_c = -45^\circ$.

2.2

- At ω_c : -3.0 dB .
- At $10\omega_c$: $20 \log_{10}(1/\sqrt{101}) = -20.0 \text{ dB}$.
- At $100\omega_c$: asymptotically -40 dB (exact -40.0).

2.3

- Same corner $\omega_c = 1/(RC) = 10 \text{ krad/s}$.
- Low-frequency asymptote: $+20 \text{ dB/decade}$ rising toward the passband (mirror of the low-pass).

3.1

- 30 dB at 4 kHz : $(\omega/\omega_c)^2 \geq 10^3 \rightarrow \omega_c \leq 2\pi \times 4000/\sqrt{1000} = 2\pi \times 4000/31.62 = 794.8 \text{ rad/s}$.
- $\leq 1 \text{ dB}$ at 400 Hz : $1/\sqrt{(1 + (\omega/\omega_c)^2)} \geq 10^{-1/20} = 0.891 \rightarrow (\omega/\omega_c)^2 \leq (1/0.891^2) - 1 = 0.259 \rightarrow \omega_c \geq 2\pi \times 400/0.509 = 4940 \text{ rad/s}$.
- The constraints conflict ($\omega_c \leq 795$ vs $\omega_c \geq 4940$): a single first-order section cannot meet both specs – report the conflict.
- Any stated corner meets only one spec; e.g. $\omega_c = 794$ gives 400 Hz loss $1/\sqrt{(1 + (2513/794)^2)} = 1/\sqrt{11} = -10.4 \text{ dB}$ – fails the passband by 9 dB .

3.2

- $\omega_0 = 1/\sqrt{25 \times 10^{-3} \times 10^{-7}} = 1/\sqrt{2.5 \times 10^{-9}} = 20000 \text{ rad/s}$.
- $Q = \omega_0 L/R = 20000 \times 0.025/50 = 10$.
- $BW = \omega_0/Q = 2000 \text{ rad/s}$; half-power at $20000 \pm 1000 = 19000$ and 21000 rad/s .
- Asymptotes: below 19000 rad/s the LC looks like an open at DC (series C), so gain rises $+20 \text{ dB/decade}$ toward the band; between corners flat 0 dB ; above 21000 rad/s the L's impedance grows – gain falls -20 dB/decade .
- At resonance $Z_L + Z_C = 0$, so the entire source appears across R: $|H| = R/R = 1 = 0 \text{ dB}$ ✓.