

4.4 Resonance

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS resonance • bandwidth • Q factor

Objective

Compute resonant frequency, bandwidth and Q for series and parallel RLC circuits; explain the impedance minimum/maximum; and verify resonant voltage/current magnifications numerically.

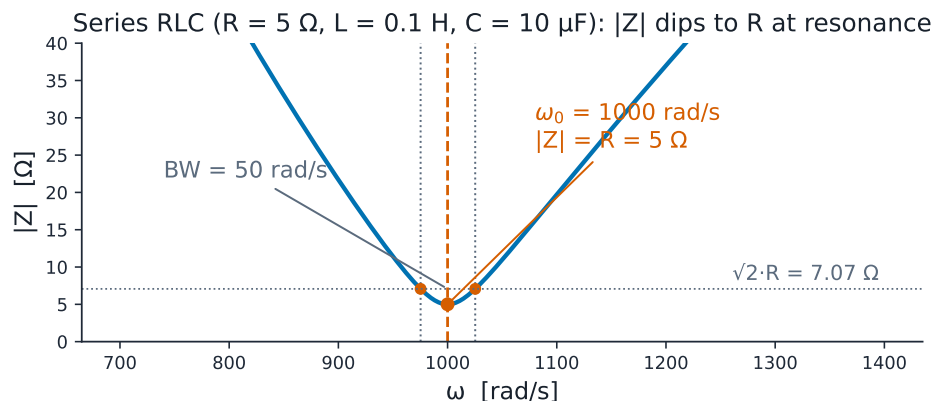
1 Key knowledge and worked examples

■ Resonance: the imaginary parts cancel

A series RLC is at **resonance** when $\omega_0 L = 1/(\omega_0 C)$:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad BW = \frac{R}{L} = \frac{\omega_0}{Q}, \quad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

At ω_0 : $Z = R$ (purely resistive, minimum magnitude) $\rightarrow$ maximum current; the **bandwidth** BW is the span between the half-power frequencies. In the PARALLEL RLC the dual holds: impedance is MAXIMUM at resonance.



$|Z|$ dips to R at ω_0 ; the half-power band is marked.

■ Worked example: voltage magnification

$R = 5 \Omega$, $L = 0.1 \text{ H}$, $C = 10 \mu\text{F}$: $\omega_0 = 1/\sqrt{10^{-6} \times 0.1} = 1000 \text{ rad/s}$, $Q = 1000 \times 0.1/5 = 20$.

At resonance with a 10 V source: $I = 10/5 = 2 \text{ A}$; $V_L = \omega_0 L I = 200 \text{ V}$ and $V_C = I/(\omega_0 C) = 200 \text{ V}$ – twenty times the source, 180° apart, cancelling each other while each alone is huge. That is what Q means, measured in volts.

■ Selectivity and the frequency sweep

A frequency sweep past resonance shows the current peaking at ω_0 and falling on both sides at $\pm BW/2$ to $1/\sqrt{2}$ of peak. High Q = narrow peak = a selective tuner; the radio tuning dial IS this arithmetic – a station spacing of $\sim 200 \text{ kHz}$ at $\sim 100 \text{ MHz}$ needs $Q \approx 500$, and $Q = \omega_0 L/R$ tells you how small R must be.

2 Practice

2.1 $L = 0.2 \text{ H}$, $C = 20 \mu\text{F}$: compute ω_0 in rad/s and Hz. [2]

2.2 A series RLC has $R = 10 \Omega$, $\omega_0 = 500 \text{ rad/s}$, $Q = 25$. Compute L, C and the bandwidth. [3]

2.3 At resonance the series circuit of 2.2 is driven by 5 V. Compute the current, V_L and V_C . [3]

3 Exam-style questions

3.1 A series RLC has $R = 2\Omega$, $L = 0.05H$, $C = 0.8\mu F$.

Compute ω_0 , Q , the bandwidth, and the half-power frequencies. With a 15 V source at resonance, compute the current and the capacitor voltage, and verify the magnification factor equals Q . Then state (with the reason) what happens to the bandwidth if R doubles with L , C unchanged. [7]

3.2 A parallel RLC ($R = 40k\Omega$ in parallel with $L = 0.4mH$ and $C = 100nF$) is driven by a 2 mA source.

Compute ω_0 and the impedance at resonance; compute the Q for the parallel circuit ($Q = R/(\omega_0 L)$) and the voltage across the tank at resonance. Verify that the tank current magnitudes in L and C each equal $Q \times$ source current, and explain why the line current is only 2 mA while the branch currents are much larger. [7]

Solutions

2.1

- $\omega_0 = 1/\sqrt{0.2 \times 2 \times 10^{-5}} = 500 \text{ rad/s}$.
- $f_0 = 500/2\pi = 79.6 \text{ Hz}$.

2.2

- $Q = \omega_0 L/R \rightarrow L = QR/\omega_0 = 25 \times 10/500 = 0.5 \text{ H}$.
- $C = 1/(\omega_0^2 L) = 1/(500^2 \times 0.5) = 8 \mu\text{F}$.
- $BW = \omega_0/Q = 500/25 = 20 \text{ rad/s}$.

2.3

- $I = 5/10 = 0.5 \text{ A}$.
- $V_L = \omega_0 LI = 500 \times 0.5 \times 0.5 = 125 \text{ V}$.
- $V_C = I/(\omega_0 C) = 0.5/(500 \times 8 \times 10^{-6}) = 125 \text{ V}$ – magnification $= 125/5 = 25 = Q$ ✓.

3.1

- $\omega_0 = 1/\sqrt{0.05 \times 0.8 \times 10^{-6}} = 5000 \text{ rad/s}$.
- $Q = \omega_0 L/R = 5000 \times 0.05/2 = 125$.
- $BW = \omega_0/Q = 40 \text{ rad/s}$; half-power at $5000 \pm 20 \text{ rad/s}$ (4980 and 5020 rad/s).
- $I = 15/2 = 7.5 \text{ A}$.
- $V_C = I/(\omega_0 C) = 7.5/(5000 \times 0.8 \times 10^{-6}) = 1875 \text{ V}$; magnification $1875/15 = 125 = Q$ ✓.
- R doubling halves Q and doubles BW ($BW = R/L$): the peak flattens – less selectivity.

3.2

- $\omega_0 = 1/\sqrt{0.4 \times 10^{-3} \times 100 \times 10^{-9}} = 1/\sqrt{4 \times 10^{-11}} = 158.1 \text{ krad/s}$ (25.1 kHz).
- At resonance the parallel tank is purely resistive: $Z = R = 40 \text{ k}\Omega$ (the L and C susceptances cancel).
- $V = I_s R = 2 \text{ mA} \times 40 \text{ k}\Omega = 80 \text{ V}$.
- $Q = R/(\omega_0 L) = 40000/(158114 \times 4 \times 10^{-4}) = 40000/63.2 = 632$.
- $I_L = V/(\omega_0 L) = 80/63.2 = 1.265 \text{ A}$ and $I_C = \omega_0 C V = 158114 \times 10^{-7} \times 80 = 1.265 \text{ A}$ – each $\approx Q \times 2 \text{ mA} = 1.264 \text{ A}$ ✓.
- The two large branch currents are equal and opposite in phase (180° apart for L vs C) – they circulate INSIDE the tank and cancel at the terminals, so the line supplies only the resistive 2 mA.