

4.2 AC power and three-phase

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS complex power • power factor
• power factor correction • three-phase

Objective

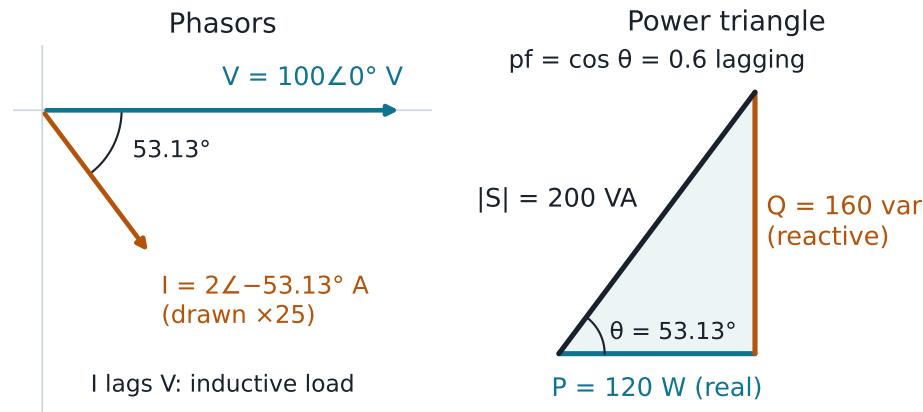
Compute real, reactive and apparent power; correct power factor with a sized capacitor; solve balanced three-phase loads in star and delta – every power answer cross-checked through I^2R and I^2X .

1 Key knowledge and worked examples

■ Complex power: $S = VI^*$

Complex power $S = VI^*$ splits into real $P = VI\cos\theta$ (W) and reactive $Q = VI\sin\theta$ (var); apparent $|S| = VI$ (VA); **power factor** $pf = \cos\theta$ (lagging for inductive loads).

Worked example. $V = 100V$, $I = 2A$, $\theta = 53.13^\circ$: $P = 120W$, $Q = 160var$, $|S| = 200 VA$, $pf = 0.6$ lagging. The independent checks: $P = |I|^2R = 4 \times 30 = 120 \checkmark$, $Q = |I|^2X = 4 \times 40 = 160 \checkmark$ – the loss-form route never touches the phase angle.



Current lags 53.13° : the power triangle P - Q - S .

■ Power-factor correction

A parallel capacitor supplies the load's reactive power locally, so the source delivers only real power. **Capacitor sizing** : $C = Q/(\omega V^2)$.

Worked example. To cancel 160 var at 100 V, 50 Hz: $C = 160/(2\pi \cdot 50 \times 100^2) = 50.9\mu F$. After correction the source current drops from 2 A to $P/V = 1.2A$ – the same real power through smaller wires, which is why utilities charge for poor power factor.

■ Three-phase: the $\sqrt{3}$ bookkeeping

Three-phase is three sources 120° apart. Star connection: $V_{line} = \sqrt{3}V_{phase}$, $I_{line} = I_{phase}$; delta: $V_{line} = V_{phase}$, $I_{line} = \sqrt{3}I_{phase}$. Either way total power $P = \sqrt{3}V_{line}I_{line}\cos\varphi$.

Worked example. A balanced star load of $20\angle 30^\circ\Omega$ per phase on a 400 V line-to-line supply: $V_{phase} = 400/\sqrt{3} = 231V$; $I_{line} = 231/20 = 11.55A$; $P = \sqrt{3} \times 400 \times 11.55 \times \cos 30^\circ = 6928W$. Check per phase: $P_{phase} = 3 \times |I|^2R = 3 \times 11.55^2 \times 17.32 = 6928W \checkmark$ (the $\sqrt{3}$ arithmetic is consistent both routes).

2 Practice

2.1 A load draws $I = 5\angle -25^\circ A$ at $V = 230\angle 0^\circ V$ (RMS). Compute P , Q , $|S|$ and the power factor. [4]

2.2 A 2 kW load at pf 0.8 lagging on a 230 V supply: find the reactive power and the capacitance that corrects the pf to unity (50 Hz). [4]

2.3 A balanced star load has $Z_{phase} = 10\angle 20^\circ \Omega$ per phase on a 415 V line-to-line supply. Find the line current and total three-phase power. [3]

3 Exam-style questions

3.1 A factory load is 300 kW at pf 0.75 lagging, supplied at 11 kV line to line, 50 Hz.

Compute the line current, the reactive power, and the capacitance per phase (star-connected correction bank) that raises the pf to 0.95 lagging. Then give the new line current and the reduction in line copper loss ($\text{loss} \propto I^2$) for the same delivered power. [8]

3.2 A balanced delta load of $Z = 30\angle 40^\circ \Omega$ per branch sits on a 400 V, three-phase supply.

Compute the phase current (in each delta branch), the line current, and the total real and reactive power. Verify the power through the per-phase loss form ($3I_{phase}^2 \times \text{Re}Z$), and state why delta lets each branch see full line voltage. [6]

Solutions

2.1

- $\theta = 25^\circ$.
- $P = 230 \times 5 \times \cos 25^\circ = 1042W$.
- $Q = 230 \times 5 \times \sin 25^\circ = 486 \text{ var}$; $|S| = 1150 \text{ VA}$; $\text{pf} = 0.906$ lagging.

2.2

- $|S| = 2000/0.8 = 2500 \text{ VA}$; $Q = |S|\sin\varphi = 2500 \times 0.6 = 1500 \text{ var}$.
- Unity correction cancels all 1500 var: $C = Q/(\omega V^2) = 1500/(2\pi \cdot 50 \times 230^2) = 90.3 \mu F$.

2.3

- $V_{\text{phase}} = 415/\sqrt{3} = 239.6V$; $I_{\text{line}} = 239.6/10 = 23.96A$.
- $P = \sqrt{3} \times 415 \times 23.96 \times \cos 20^\circ = 16.18kW$.

3.1

- $\varphi = \arccos 0.75 = 41.41^\circ$.
- $I = P/(\sqrt{3}V_L\text{pf}) = 300000/(\sqrt{3} \times 11000 \times 0.75) = 20.99A$.
- $Q = P\tan\varphi = 300000 \times 0.8819 = 264.6kvar$.
- Target pf 0.95: $\varphi' = 18.19^\circ$, $Q' = 300000 \times \tan 18.19^\circ = 98.6 \text{ kvar}$; the bank supplies $264.6 - 98.6 = 166.0 \text{ kvar}$.
- Per phase: 55.32 kvar at $V_{\text{phase}} = 11000/\sqrt{3} = 6351 \text{ V}$: $C = 55320/(2\pi \cdot 50 \times 6351^2) = 4.37\mu F$ per phase (high voltage means small capacitance).
- New current: $I' = 300000/(\sqrt{3} \times 11000 \times 0.95) = 16.57A$.
- Loss ratio $(I'/I)^2 = (16.57/20.99)^2 = 0.623$ – a 37.7% line-loss reduction.

3.2

- Delta: each branch sees $V_{\text{phase}} = V_{\text{line}} = 400V$.
- $I_{\text{phase}} = 400/30 = 13.33A$.
- $I_{\text{line}} = \sqrt{3} \times 13.33 = 23.09A$.
- $P = \sqrt{3} \times 400 \times 23.09 \times \cos 40^\circ = 12.26 \text{ kW}$; $Q = \sqrt{3} \times 400 \times 23.09 \times \sin 40^\circ = 10.28 \text{ kvar}$.
- Check: $3 \times I_{\text{phase}}^2 \times \text{Re}Z = 3 \times 177.8 \times 22.98 = 12.26 \text{ kW}$ ✓.
- Delta connects each branch across line pairs – full line voltage per branch, which is why $I_{\text{line}} = \sqrt{3}I_{\text{phase}}$ (two branches feed each line).