

4.1 Sinusoidal steady state and phasors

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS phasor • impedance

Objective

Convert sinusoids to phasors, compute impedances, and solve AC circuits with all the DC methods – every answer verified in both polar and rectangular form.

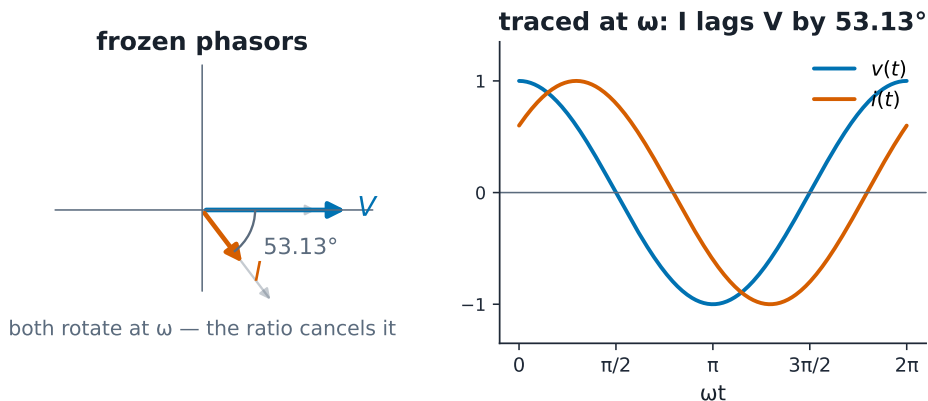
1 Key knowledge and worked examples

■ The phasor transformation

$v(t) = V_m \cos(\omega t + \varphi)$ becomes the **phasor** $V = V_m \angle \varphi$. At one frequency, every element is an **impedance** :

- $Z_R = R \angle 0^\circ$, $Z_L = j\omega L \angle +90^\circ$, $Z_C = 1/(j\omega C) \angle -90^\circ$.

Impedances combine exactly like resistors: series adds, parallel is product-over-sum. Ohm's law, dividers, node and mesh analysis all carry over.



The rotating picture behind the frozen phasors: ratios cancel the rotation.

■ Worked example: the RL load, both forms

$R = 30\Omega$ in series with $\omega L = 40\Omega$, driven by $V = 100\angle 0^\circ V$.

- Polar: $Z = \sqrt{30^2 + 40^2} \angle \text{atan}(40/30) = 50\angle 53.13^\circ$; $I = 100/50\angle(0 - 53.13^\circ) = 2\angle -53.13^\circ A$.
- Rectangular check: $I = 100(30 - j40)/(30^2 + 40^2) = (3000 - j4000)/2500 = 1.2 - j1.6 A$; $|I| = 2 \checkmark$ and $\text{atan2}(-1.6, 1.2) = -53.13^\circ \checkmark$.

Current lags voltage by 53.13° – the inductor's mark, visible in the phasor before any arithmetic on the time domain.

■ Time-domain round trip

$i(t) = 2\sqrt{2}\cos(\omega t - 53.13^\circ) A$ if the phasor used RMS amplitude convention $V = 100V$ RMS – state which convention you use and stay in it. The final check is dimensional: the phasor method predicts the measured amplitude AND phase of every branch voltage, which is what an oscilloscope shows.

2 Practice

2.1 Convert to polar: $Z = 8 + j15$, $Z = 12 - j5$. [2]

2.2 A 40Ω resistor in series with a capacitor whose reactance is 30Ω at the operating frequency: find Z , and I for $V = 80\angle 0^\circ V$. [3]

2.3 Two impedances $Z_1 = 6 + j8$ and $Z_2 = 12 - j4$ are in parallel. Find the equivalent impedance (verify by computing $Z_1 Z_2 / (Z_1 + Z_2)$ both ways). [4]

3 Exam-style questions

3.1 A circuit has $R = 10\Omega$ in series with L ($\omega L = 20\ \Omega$) and C (reactance $10\ \Omega$ at the same ω), driven by $V_s = 60\angle 30^\circ V$.

Compute the total impedance, the current phasor, and the voltage across the capacitor (as a phasor, polar form). Verify the loop: the sum of the three element voltage phasors returns V_s . [7]

3.2 A phasor-domain nodal analysis: node A is fed by $V_s = 50\angle 0^\circ V$ through $Z_1 = j10\Omega$, and connected to ground through $Z_2 = -j10\Omega$ (a capacitor at this frequency) and, in parallel with it, $Z_3 = 5\Omega$.

Write the single node equation, solve for V_A , and compute the current in each of the three branches. Verify KCL at node A. [6]

Solutions

2.1

- $Z_1 = 17\angle 61.93^\circ$.
- $Z_2 = 13\angle -22.62^\circ$.

2.2

- $Z = 40 - j30 = 50\angle -36.87^\circ\Omega$.
- $I = 80/50\angle(0 + 36.87^\circ) = 1.6\angle 36.87^\circ A$.

2.3

- Product-over-sum: $Z_1 Z_2 = (6 + j8)(12 - j4) = 104 + j72$; $Z_1 + Z_2 = 18 + j4$.
- Rationalise: $(104 + j72)(18 - j4) = (104 \cdot 18 + 72 \cdot 4) + j(72 \cdot 18 - 104 \cdot 4) = 2160 + j880$, over $18^2 + 4^2 = 340$.
- $Z = (2160 + j880)/340 = 6.35 + j2.59\Omega (= 6.86\angle 22.17^\circ)$.
- Rectangular check: $|Z| = \sqrt{40.3 + 6.7} = 6.86 \checkmark$.

3.1

- $Z = 10 + j20 - j10 = 10 + j10 = 14.14\angle 45^\circ\Omega$.
- $I = 60/14.14\angle(30 - 45) = 4.243\angle -15^\circ A$.
- $V_C = I \times (-j10) = 42.43\angle -105^\circ V$.
- Loop check: $V_R = 42.43\angle -15^\circ$, $V_L = 84.85\angle 75^\circ$; sum $V_R + V_L + V_C = 42.43\angle -15^\circ + 84.85\angle 75^\circ + 42.43\angle -105^\circ$ – rectangular: $(40.95 - j10.97) + (21.97 + j81.96) + (-10.97 - j40.95) = 51.95 + j30.04 \approx 60\angle 30^\circ \checkmark$.

3.2

- Node equation: $(V_A - V_s)/(j10) + V_A/(-j10) + V_A/5 = 0$.
- Multiply by $j10$: $V_A - 50 - V_A + j2 \cdot V_A = 0 \rightarrow j2V_A = 50 \rightarrow V_A = 50/(j2) = -j25 = 25\angle -90^\circ V$.
- $I_1 = (V_A - V_s)/(j10) = (-j25 - 50)/(j10) = (-50 - j25)/(j10)$. Multiply numerator and denominator by $-j$: $(-j)(-50 - j25)/(10) = (j50 - 25)/10 = -2.5 + j5 = 5.59\angle 116.57^\circ A$.
- $I_2 = V_A/(-j10) = (-j25)/(-j10) = 2.5 = 2.5\angle 0^\circ A$.
- $I_3 = V_A/5 = -j5 = 5\angle -90^\circ A$.
- KCL (all currents leaving node A): $I_1 + I_2 + I_3 = (-2.5 + j5) + 2.5 + (-j5) = 0 \checkmark$ – note I_1 is defined TOWARD the source, so it leaves the node.