UNIVERSITY ELECTRICAL ENGINEERING • EXERCISE SHEET

3.3 Combinational building blocks

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS multiplexer • decoder • full adder • hazard

Objective

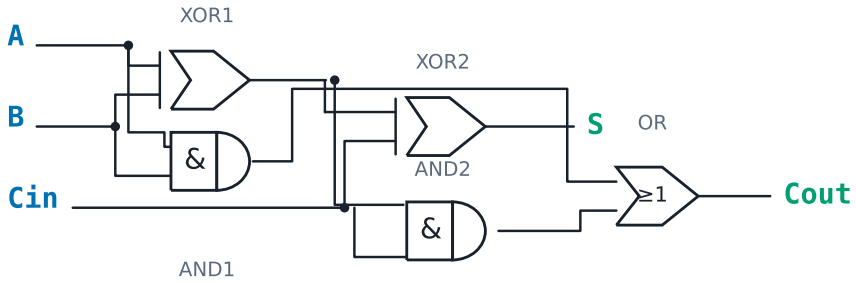
Design with multiplexers, decoders and adders; read a select code to a data line; explain hazards and why redundant grouping covers them – every block answer verified by evaluating all input rows.

1 Key knowledge and worked examples

■ **Three blocks cover most designs**

- **Multiplexer** (MUX): 2^n data inputs, n **select** lines; $Y = D_i$ where i is the select code's value. A 4-to-1 MUX with `S1S0 = 10` outputs `D2`.
- **Decoder** : n inputs, 2^n outputs, exactly one high – output i is the minterm i. Any SOP function = OR of decoder outputs (plus NOT for the variable complements is unnecessary – the decoder generates all minterms).
- **Full adder** : $S = A \oplus B \oplus Cin$, $Cout = AB + Cin \cdot (A \oplus B)$. Ripple n of them for an n-bit adder.

The full adder: five gates, three inputs, two outputs



■ **Worked example: full adder, all eight rows**

At $A = 1, B = 1, Cin = 1$: $S = 1 \oplus 1 \oplus 1 = 1$; $Cout = 1 \cdot 1 + 1 \cdot (1 \oplus 1) = 1$. Three ones in → binary `11` out: $2 \cdot 1 + 1 = 3$ ✓. The invariant $2 \cdot Cout + S = A + B + Cin$ holds on all eight rows – that is the exhaustive verification, and it is how the HDL testbench of unit 3.7 checks the same block.

■ **Hazards: unequal paths glitch**

A **hazard** is a momentary false output when one input change reaches the output through paths of different delays. The classic: `F = AB + A'C` – when B = C = 1 and A changes 1→0, both terms can be momentarily 0. The remedy is the redundant **consensus** term `BC` (adding it changes no truth-table value but holds the output during the switch). Hazards matter because the glitch can downstream be captured as a clock edge – the reason synchronous designs sample on edges, not levels.

2 Practice

2.1 An 8-to-1 MUX has data inputs `D0..D7` and selects `S2S1S0 = 101`. Which input reaches the output? What is `S2S1S0` to select `D6`? [3]

2.2 A 3-to-8 decoder has inputs `A2A1A0 = 011`. Which output is high? Which minterm does it represent? [3]

2.3 Compute a full adder at $A = 0, B = 1, Cin = 1$: give S and Cout, and verify with the invariant. [3]

3 Exam-style questions

3.1 Design a 4-bit ripple adder for `A + B` where $A = 1011$ and $B = 0110$.

Work each full adder stage (LSB first), giving S and Cout per stage, the final 4-bit sum and the carry out. Verify in decimal, and state the largest pair of 4-bit unsigned values that does NOT produce a carry out. [6]

3.2 A circuit must output 1 when a 3-bit unsigned input is greater than 5.

Realise `F` as an OR of 3-to-8 decoder outputs (name the minterms), then minimise on a Karnaugh map and give the minimal SOP. Verify by evaluating your expression on all 8 inputs, and state which single-variable don't-care condition could NOT be used here (and why there are no don't-cares at all). [6]

Solutions

2.1

- Select value $101_2 = 5 \rightarrow$ `D5` reaches the output.
- To select `D6`: `S2S1S0 = 110`.

2.2

- $011_2 = 3 \rightarrow$ output `Y3` is high (all others low).
- It represents minterm 3: $A2' \cdot A1 \cdot A0$.

2.3

- $S = 0 \oplus 1 \oplus 1 = 0$.
- $Cout = 0 \cdot 1 + 1 \cdot (0 \oplus 1) = 1$.
- Invariant: $2 \cdot 1 + 0 = 2 = 0 + 1 + 1$ ✓.

3.1

- Stage 0: `1+0+0`: S0=1, C1=0.
- Stage 1: `1+1+0`: S1=0, C2=1.
- Stage 2: `0+1+1`: S2=0, C3=1.
- Stage 3: `1+0+1`: S3=0, Cout=1.
- Sum $S = 0001$, Cout $1 \rightarrow$ the 5-bit result `10001 = 17 = 11 + 6` ✓.
- No carry out requires $A + B \leq 15$; the largest pair is any with sum 15, e.g. `15 + 0` or `1111 + 0000` (equivalently `1000 + 0111`).

3.2

- Greater than 5 (of 0–7): minterms 6, 7 $\rightarrow F = Y6 + Y7$.
- Karnaugh map: cells 6 (110), 7 (111) adjacent $\rightarrow$ group $= A \cdot B$ (C eliminated).
- Minimal SOP: `F = AB`.
- Verify: $AB = 1$ exactly on inputs 6, 7 ✓ (all eight rows).
- No don't-cares exist because every 3-bit pattern is a real, specified input – the function's value matters on all 8 rows; don't-cares require impossible or unspecified combinations, and none are.