2.2 Complex numbers and phasor algebra

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS complex number • modulus • argument • phasor • complex conjugate

Objective

Convert fluently between rectangular and polar forms, multiply and divide in polar form, and check each result by reversing the operation or evaluating both forms at the same angle.

1 Key knowledge and worked examples

■ Two forms, one number

A **complex number** `z = a + jb` has **modulus** $r = \sqrt{a^2 + b^2}$ and **argument** $\theta = atan2(b, a)$:

$$a + jb = r\, e^{j\theta} \quad \text{(Euler)}$$

Euler's identity is the bridge EE lives on: $e^{j\theta} = \cos\theta + j\sin\theta$. A **phasor** is just a complex amplitude – sinusoids become arithmetic.

■ Polar arithmetic: multiply and divide

- Multiply: moduli multiply, arguments ADD – $r_1 r_2 e^{j(\theta_1+\theta_2)}$.
- Divide: moduli divide, arguments SUBTRACT.

Worked example. $(6\angle 30°)/(2\angle 80°) = 3\angle -50°$ – read off directly. The inverse check: $3\angle -50° \times 2\angle 80° = 6\angle 30°$, back to the start ✓. In rectangular form the same division needs rationalising `(5.196 + j3)/(0.347 + j1.970)` – correct but slow, which is why AC analysis is done in polar.

■ **The conversion checks**

Rectangular → polar → rectangular must return the start: `3 + j4 = 5∠53.13° = 5(cos 53.13° + j sin 53.13°) = 3.000 + j4.000` ✓. And two facts worth having by heart: $1/j = -j$ (a 90° clockwise rotation) and $|z|^2 = z \cdot z*$ – the **complex conjugate** `z* = a - jb` turns division into multiplication.

2 Practice

2.1 Convert to polar: `3 + j4`, `-2 + j2`, `-5 - j12` (moduli to 2 dp, arguments in degrees to 1 dp). [3]

2.2 Compute in polar form: $(8\angle 15°) \times (2\angle 40°)$ and $(8\angle 15°)/(2\angle 40°)$. [2]

2.3 Evaluate `1/j`, j^3, and $|3 + j4|^2$ using the conjugate rule. [3]

3 Exam-style questions

3.1 An impedance $Z_1 = 4 + j3\Omega$ carries a phasor current $I = 2\angle 20°A$.

Convert Z_1 to polar form, compute the voltage phasor $V = Z_1 I$ in polar form, and convert `V` back to rectangular. Verify the result by computing $Z_1 \cdot I$ wholly in rectangular form (state both to 2 dp). [6]

3.2 Show algebraically that $|z|^2 = z \cdot z*$ for `z = a + jb`, and use it to compute `1/(3 + j4)` exactly as `(3 - j4)/25`.

Then evaluate `1/(3 + j4)` in polar form ($1\angle 0° \div 5\angle 53.13°$) and confirm both routes agree to 2 dp. [6]

Solutions

2.1

- $3 + j4 = 5\angle 53.1°$.
- $-2 + j2 = 2.83\angle 135°$.
- $-5 - j12 = 13\angle -112.6°$.

2.2

- $16\angle 55°$.
- $4\angle -25°$.

2.3

- $1/j = -j$.
- $j^3 = -j$.
- $|3 + j4|^2 = (3 + j4)(3 - j4) = 9 + 16 = 25$.

3.1

- $Z_1 = 5\angle 36.87°\Omega$.
- $V = Z_1 I = 5\angle 36.87° \times 2\angle 20° = 10\angle 56.87° V$.
- Rectangular: `10(cos 56.87° + j sin 56.87°) = 5.46 + j8.36 V`.
- Rectangular route: `(4 + j3)(1.879 + j0.684) = 7.516 + j2.736 + j5.637 + j`^{2}`2.052 = 5.464 + j8.373` ✓ (agreeing to rounding).

3.2

- $z \cdot z* = (a + jb)(a - jb) = a^2 - (jb)^2 = a^2 + b^2 = |z|^2$.
- `1/(3+j4) = (3-j4)/((3+j4)(3-j4)) = (3-j4)/25 = 0.12 - j0.16`.
- Polar route: $1\angle 0° \div 5\angle 53.13° = 0.2\angle -53.13°$.
- `0.2(cos -53.13° + j sin -53.13°) = 0.12 - j0.16` ✓ – exact agreement.