2.1 Calculus review for signals

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS derivative • integral • rectangular pulse

Objective

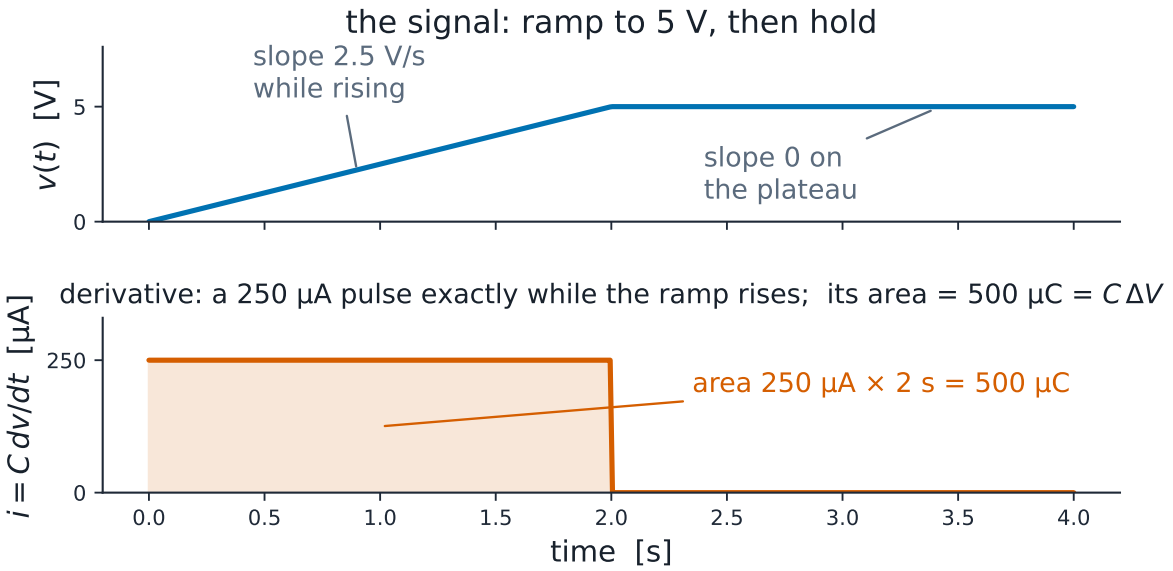
Read derivatives and integrals directly off signals, use the exponential's self-similarity, and check every calculus answer by a second method (area, units, or a capacitor identity).

1 Key knowledge and worked examples

■ Slope and area, on signals

The **derivative** `dv/dt` is the slope; the **integral** $\int i dt$ is the accumulated area. Three facts do most of the EE work:

- $d/dt(e^{at}) = a \cdot e^{at}$ – exponentials are their own slopes.
- A **rectangular pulse** of height `A`, width τ has area $A\tau$.
- A triangle's area is half its enclosing rectangle.



■ **Worked example: the ramp-charged capacitor**

`v(t) = 2.5t` volts for 2 s, $C = 100\mu F$:

- Current: $i = C dv/dt = 100\mu \times 2.5 = 250\mu A$ – constant, because the ramp's slope is constant.
- Charge: $\int i dt = 250\mu \times 2 = 500\mu C$.
- The check: $Q = C \cdot \Delta V = 100\mu \times 5 = 500\mu C$ – the capacitor identity closes the loop with a completely different method.

Two routes, one answer: slope-plus-area against the device law. That habit is this unit.

■ **Units are a free check**

`[C dv/dt]` = farads × volts/second = coulombs/second = amperes ✓. An integral over time carries "per second" × "seconds" = the plain unit – so $\int i dt$ in coulombs passes, while a result in amperes would convict the working. Every EE formula can be tried this way in ten seconds.

2 Practice

2.1 Differentiate each: $v = 5t^3$, $i = 4e^{-2t}$, $p = 7$ (constant). [3]

2.2 Compute $\int_0^3 (2t) dt$ and state its units if `2t` is a current in milliamps with `t` in seconds. [3]

2.3 A voltage ramps $0 \to 12$ V in 4 s into a 220 µF capacitor. Find `dv/dt`, the current, and verify the charge two ways. [4]

3 Exam-style questions

3.1 A current pulse `i(t) = 50 mA` flows for 3 ms into a capacitor, then stops. The capacitor is 47 µF.

Find the charge delivered and the voltage change (two methods for the charge: integral and pulse area). Then compute the capacitor's stored-energy change and check its units reduce to joules. [6]

3.2 A signal $v(t) = 8e^{-3t}$ V for $t \geq 0$ decays through a resistor, so `i = v/R` with $R = 2k\Omega$.

Write `i(t)`, evaluate `i(0)` and `i(0.5)`, and compute the total charge $\int_0^\infty i dt$ (use $\int e^{-at}dt = 1/a$). Verify the total charge also equals the initial current divided by the decay rate. [6]

Solutions

2.1

- $dv/dt = 15t^2$.
- $di/dt = -8e^{-2t}$.
- $dp/dt = 0$.

2.2

- $\int_0^3 2t dt = t^2|_0^3 = 9$.
- Units: mA × s = microcoulombs... careful: milliamps × seconds = millicoulombs – 9 mC.

2.3

- $dv/dt = 12/4 = 3V/s$.
- $i = C dv/dt = 220\mu \times 3 = 660\mu A$.
- Charge way 1: $\int i dt = 660\mu \times 4 = 2.64mC$.
- Way 2: $C \cdot \Delta V = 220\mu \times 12 = 2.64mC$ ✓.

3.1

- Charge by integral: $\int_0^{3ms} 50mA dt = 50m \times 3m = 150\mu C$.
- By pulse area: height × width = the same 150 µC ✓.
- Voltage change: $\Delta V = Q/C = 150\mu/47\mu = 3.19V$.
- Energy: $\Delta W = ½C(V_0 + \Delta V)^2 - ½CV_0^2$; from 0, $= ½ \times 47\mu \times 3.19^2 = 239\mu J$.
- Units: $F \cdot V^2 = (C/V) \cdot V^2 = C \cdot V = C \cdot (J/C) = J$ ✓.

3.2

- $i(t) = v/R = 4e^{-3t}mA$.
- `i(0) = 4 mA`; $i(0.5) = 4e^{-1.5} = 0.892mA$.
- $\int_0^\infty i dt = 4m \times (1/3) = 1.333mC$.
- Check: initial current / rate = $4mA/3s^{-1} = 1.333mC$ ✓ – the exponential's area is height times time-constant.