12.7 Signal-processing project

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS processing gain • resolution

Objective

Turn a specification into a verified pipeline and report measured margins honestly.

1 Key knowledge and worked examples

■ The method

Specify first (tone detector: 1 Hz bins, under 1 s latency, detect at −10 dB SNR); verify each stage alone (filter → window → FFT → threshold); report spec, measured and margin per line.

■ Worked example: the spec bites back

1 Hz bins at $f_s = 1$ kHz needs $N \geq 1000$ – already 1.0 s against the "<1 s" line. $N = 1024$: 0.977 Hz ✓, 1.024 s ×; $N = 1000$: exactly on the line. The margin table records the trade ✓.

■ The feasibility number

Processing gain $10 \log_{10}(N/2)$: 27 dB at $N = 1024$ – a −10 dB SNR tone becomes +17 dB in the decision bin ✓ detectable before anything is built.

2 Practice

2.1 Compute the bin processing gain at $N = 256$. [3]

2.2 A spec demands 0.5 Hz bins at $f_s = 2$ kHz. Minimum record length? [3]

2.3 Why is "tuning the detector until the lab tone is found" a defect rather than a method? [3]

3 Exam-style questions

3.1 A tone detector must find a 440 Hz tone within ±2 Hz at $f_s = 4$ kHz, reporting within 0.5 s, at −6 dB SNR.

Size N (power of two) and check the latency line. Compute the bin number of 440 Hz and the quantisation error in Hz. State the processing gain and the decision-bin SNR. State one spec line this design misses (or exactly meets) and its margin. [9]

3.2 The project report of a rival team claims “meets all specs” with a 6-line table where two rows read “PASS” but show no measured value.

Give the three defects of this table against the honest method. Compute what the missing numbers would need to show for a 1 Hz-bin spec with $N = 900$ at $f_s = 1$ kHz. State the single change that turns the table honest. [7]

Solutions

2.1

- $10 \log_{10}(128) = 21.1$ dB.

2.2

- $\Delta f = f_s/N \leq 0.5$ Hz $\rightarrow N \geq 4000 \rightarrow 2.0$ s of data.

2.3

- It tunes to the test signal (overfitting): the detector passes the lab tone and fails the world – measurement, not memorisation, is the method.

3.1

- $\Delta f \leq 2$ Hz $\rightarrow N \geq 2000 \rightarrow N = 2048$ (power of two): $\Delta f = 1.953$ Hz ✓.
- Latency: 2048 samples / 4000 Hz = 0.512 s – misses the 0.5 s line by 12 ms × (the trade the margin table must show).
- Bin: $440/1.953 = 225.28 \rightarrow$ bin 225, exact frequency 439.45 Hz, quantisation error 0.55 Hz ✓ inside ±2.
- Processing gain $10 \log_{10}(1024) = 30.1$ dB $\rightarrow$ decision-bin SNR $-6 + 30.1 = 24.1$ dB ✓.

3.2

- Defects: no measured values (PASS is a claim, not a number); no margins (nothing to audit); silent goals (lines measured in code but not reported).
- $N = 900$ at 1 kHz: $\Delta f = 1.111$ Hz – the 1 Hz line FAILS with a -0.111 Hz margin, which the table must show.
- Change: every row lists spec, measured value, and signed margin – no unnumbered PASS.