11.6 Digital modulation

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS constellation

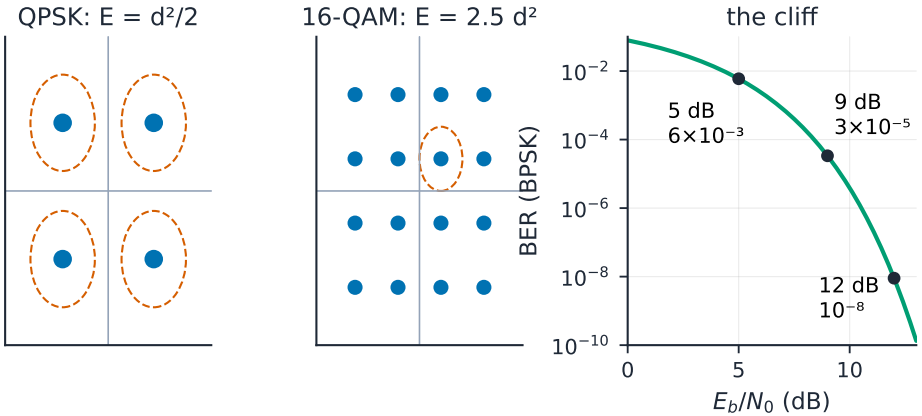
Objective

Derive constellations for ASK, PSK and QAM, compare their error performance under AWGN, and match bit rate, bandwidth and energy in a link design.

1 Key knowledge and worked examples

■ The constellation plane

I/Q axes; ASK moves radially, PSK rotates, QAM does both. Bits per symbol $log_2 M$: QPSK 2, 16-QAM 4, 64-QAM 6 – spectral efficiency scales with $log_2 M$, noise tolerance falls.



Equal minimum distance d: 16-QAM packs 2× the bits per symbol for 5× the average energy.

■ Worked example: energy accounting

Equal minimum distance d = 2: QPSK average symbol energy 2 (points at (±1, ±1)); 16-QAM average 10 (grid ±1, ±3) – a factor 5 = 7.0 dB more energy for 2× the bits per symbol ✓.

■ The cliff

BPSK: $BER = Q(\sqrt{2E_b/N_0})$ – 5 dB gives 6×10^{-3}, 9 dB gives 3×10^{-5}, 12 dB reaches 9×10^{-9}: each dB buys an order of magnitude near the wall.

2 Practice

2.1 How many bits per symbol does 64-QAM carry? [3]

2.2 Compute BPSK's BER at $E_b/N_0 = 8$ dB (Q of 3.55 is 1.9×10^{-4} – verify the argument first). [3]

2.3 For equal minimum distance, how much more average symbol energy does 16-QAM need than QPSK (in dB)? [3]

3 Exam-style questions

3.1 A 2 Mbps link may use QPSK or 16-QAM with raised-cosine shaping ($\alpha = 0.35$ available).

Compute the symbol rate for each scheme. Compute the required bandwidth for each. Compute each scheme's spectral efficiency (bps/Hz) and verify against the Shannon capacity of that bandwidth at 20 dB SNR. State the energy cost of choosing 16-QAM. [8]

3.2 A satellite receiver delivers $E_b/N_0 = 5$ dB to a BPSK link.

Compute the BER. The operator raises power by 4 dB: compute the new E_b/N_0 and BER. A switch to QPSK at the SAME symbol energy and rate halves E_b (two bits share the symbol): compute the resulting E_b/N_0 and BER, and state the design lesson about bits, symbols and energy. [7]

Solutions

2.1

- $log_2 64 = 6$ bits/symbol.

2.2

- $E_b/N_0 = 10^{0.8} = 6.31$; $\sqrt{2 \times 6.31} = 3.55$ ✓.
- $BER = Q(3.55) = 1.9 \times 10^{-4}$.

2.3

- Ratio $10/2 = 5 \rightarrow 10log_{10}5 = 7.0dB$.

3.1

- QPSK: $R_s = 2M/2 = 1Msym/s$; 16-QAM: $R_s = 2M/4 = 0.5Msym/s$.
- Bandwidths: `0.675 MHz` and `0.3375 MHz` ($B = (1+\alpha)R_s/2$).
- Efficiency: $2/0.675 = 2.96$ bps/Hz; $2/0.3375 = 5.93$ bps/Hz.
- Shannon at 20 dB: $log_2(101) = 6.66bps/Hz$ – both fit under the ceiling ($5.93 < 6.66$ ✓, with little headroom for 16-QAM).
- Energy cost: 16-QAM needs ≈ 7 dB more symbol energy than QPSK at equal error rate – bandwidth saved is power spent.

3.2

- At 5 dB: BER $= Q(\sqrt{2 \times 3.16}) = Q(2.51) = 6.0 \times 10^{-3}$.
- +4 dB $\rightarrow$ 9 dB (7.94): $Q(\sqrt{15.9}) = Q(3.99) = 3.3 \times 10^{-5}$.
- QPSK at same symbol energy: E_b halves $\rightarrow$ E_b/N_0 drops 3 dB to 2 dB (1.58): $Q(\sqrt{3.16}) = Q(1.78) = 3.7 \times 10^{-2}$.
- Lesson: bits, not symbols, consume energy – packing two bits into one symbol halves the energy per bit unless power rises; error rate depends on E_b, never on how prettily the bits are arranged.