

11.3 Angle modulation (FM/PM)

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS frequency deviation • Carson's rule

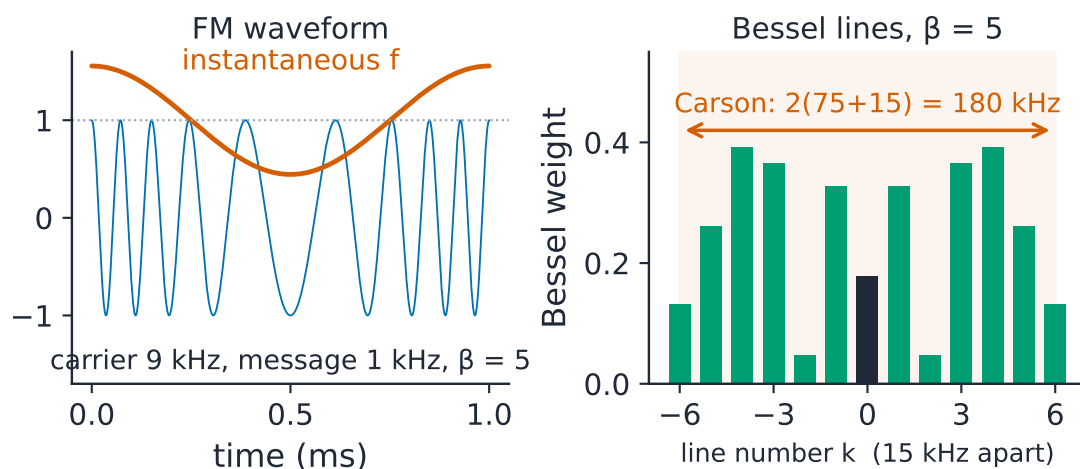
Objective

Derive instantaneous frequency and Carson bandwidth for FM and PM, tell narrowband from wideband, and explain capture and threshold qualitatively.

1 Key knowledge and worked examples

■ The two angles

FM: $f_i = f_c + k_f \cdot x(t)$, deviation Δf , $\beta = \Delta f / f_m$. PM: phase $k_p \cdot x(t)$, $\beta = k_p \cdot A_m$ – frequency-independent.



The message bends the instantaneous frequency; Carson's rule fences the spectral spread.

■ Carson's rule

$B \approx 2(\Delta f + f_m) = 2f_m(1 + \beta)$: broadcast's $\Delta f = 75 \text{ kHz}$, $f_m = 15 \text{ kHz}$ gives $\beta = 5$ and $B = 180 \text{ kHz}$ inside a 200 kHz channel ✓.

■ Narrow vs wide

$\beta \ll 1$: $B \approx 2f_m$ (narrowband, walkie-talkie). $\beta \gg 1$: $B \approx 2\Delta f$, and the post-detector SNR grows as $3\beta^2/2$ – bandwidth bought with signal-to-noise (unit 11.4).

2 Practice

2.1 Compute β and Carson bandwidth for $\Delta f = 40 \text{ kHz}$, $f_m = 8 \text{ kHz}$. [3]

2.2 A narrowband system runs $\beta = 0.15$ at $f_m = 4 \text{ kHz}$. Estimate its bandwidth. [3]

2.3 A PM modulator has $k_p = 2.5 \text{ rad/V}$ and the message amplitude is 2 V. Compute β . [3]

3 Exam-style questions

3.1 An FM link uses $\Delta f = 50 \text{ kHz}$ with messages up to 15 kHz.

Compute β and Carson's bandwidth at the top message frequency. Compute B for a 300 Hz tone at the same deviation, and comment on the ratio. The channel is 150 kHz wide: state whether the design fits at both message frequencies. Explain why FM's bandwidth is set by the WORST (highest) modulating frequency. [8]

3.2 Compare FM and PM for a message whose amplitude is constant but whose frequency varies from 100 Hz to 10 kHz, with Δf (FM) = 40 kHz and k_p (PM) chosen to give $\beta = 4$ at 1 kHz.

Compute FM's β at 100 Hz and at 10 kHz. Compute PM's β at the same two frequencies. State which system keeps its deviation (and bandwidth) sensible across the range and which needs pre-emphasis, and explain the threshold/capture behaviour a wideband receiver shows near the FM threshold. [7]

Solutions

2.1

- $\beta = 40/8 = 5$.
- $B = 2(40 + 8) = 96\text{kHz}$.

2.2

- $B \approx 2f_m = 8\text{kHz}$ (Carson gives $2 \times 4 \times 1.15 = 9.2\text{kHz}$).

2.3

- $\beta = 2.5 \times 2 = 5$ – amplitude only, no frequency dependence.

3.1

- At 15 kHz: $\beta = 50/15 = 3.33$, $B = 2 \times 65 = 130\text{kHz}$.
- At 300 Hz: $\beta = 50/0.3 = 167$, $B = 2 \times 50.3 \approx 101\text{kHz}$.
- Ratio: B varies only $130/101 = 1.3\times$ while β varies $50\times$ – at high f_m the deviation term dominates, at low f_m the message term does; B never falls below $2\Delta f$.
- 150 kHz channel: fits both ($130 \leq 150$, $101 \leq 150$) ✓.
- Highest f_m maximizes $2f_m(1 + \beta) = 2(\Delta f + f_m)$: Δf is fixed by the modulator, so the top message frequency sets the worst-case width.

3.2

- FM: $\beta = \Delta f / f_m = 40/0.1 = 400$ at 100 Hz; $40/10 = 4$ at 10 kHz.
- PM: $\beta \propto$ amplitude only – constant at 4 across the whole range (k_p fixed; the 1 kHz calibration is irrelevant).
- FM's β swings 100:1 – its deviation is extreme at low frequencies, so FM needs pre-emphasis (boost high message frequencies before modulation) to level the spectrum; PM is naturally flat.
- Near the FM threshold (input SNR below roughly 10 dB), noise peaks capture the limiter and the detector emits loud clicks; above it, the stronger signal completely suppresses the weaker one – the capture effect that makes FM sound clean or broken, never in between.