

11.1 Signals, spectra and bandwidth

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS bandwidth • sideband

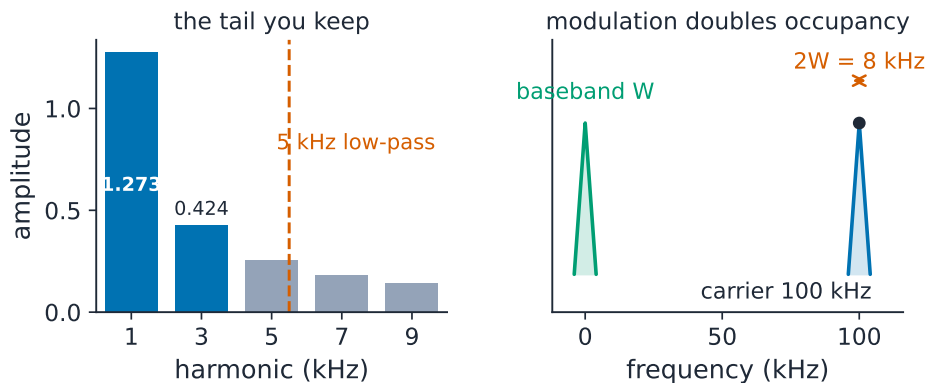
Objective

Compute the spectra of standard signals and modulated waveforms, compare bandwidth measures, and fit signals to channel constraints.

1 Key knowledge and worked examples

■ Harmonic structure

A square wave carries odd harmonics at $4/(\pi k)$ of the fundamental: 1st at 1.273, 3rd at 0.424, 5th at 0.255 – and 81% of the power sits in the fundamental ($8/\pi^2 = 0.811$), the Parseval intuition.



The tail you keep is the bandwidth you pay for; modulation doubles the occupancy.

■ Worked example: truncation

A 1 kHz square wave through a 5 kHz low-pass keeps the 1st and 3rd harmonics; the 5th (at 5 kHz, amplitude 0.255) is the boundary case – the reconstructed wave rounds over exactly as Fourier predicts.

■ Modulation shifts, never reshapes

A baseband signal of width W mixed onto a carrier occupies $2W$ of radio spectrum (both sidebands); SSB halves it by sending one. A 4 kHz message on a 100 kHz carrier: 96–104 kHz.

2 Practice

2.1 Compute the 3rd- and 5th-harmonic amplitudes (relative to the fundamental) of a square wave. [3]

2.2 A 2 kHz square wave passes a 4 kHz low-pass filter. Which harmonics survive, and what does the output look like? [3]

2.3 A telephone channel passes 300–3400 Hz. Compute its bandwidth and the minimum sampling rate Nyquist demands. [3]

3 Exam-style questions

3.1 A 1 kHz square wave feeds a low-pass channel.

Give the amplitudes of the first three odd harmonics (relative to a unit fundamental). Compute what fraction of the total power the fundamental alone carries, stating the series identity you use. The channel cuts at 4 kHz: state which harmonics pass and sketch-describe the output waveform. Explain why “bandwidth” for this signal is a design choice rather than a single number. [8]

3.2 A message occupying 0–4 kHz amplitude-modulates a 100 kHz carrier (conventional double-sideband AM).

Draw/describe the spectrum: give the frequency ranges of the upper and lower sidebands and the total occupied bandwidth. Repeat for single-sideband (upper sideband only). If the channel assignment is exactly 8 kHz wide, state which scheme fits and what it costs. [7]

Solutions

2.1

- 3rd: $4/(3\pi) = 0.424$ of the fundamental.
- 5th: $4/(5\pi) = 0.255$.

2.2

- Fundamental 2 kHz passes; 3rd harmonic at 6 kHz (and above) is cut.
- Output = a clean 2 kHz sine – an ideal cut removes everything but the fundamental; a real filter's finite roll-off leaves a faint 6 kHz vestige.

2.3

- Bandwidth $3400 - 300 = 3100$ Hz.
- Nyquist needs $f_s > 2 \times 3400 = 6.8\text{kHz}$; the standard 8 kHz sampling leaves guard space above the band.

3.1

- Amplitudes 1.273, 0.424, 0.255 (unit fundamental $4/\pi$).
- Power fraction $= 1/\sum (1/k^2)$ over odd $k = 8/\pi^2 = 0.811$ (using $\sum_{\text{odd}} 1/k^2 = \pi^2/8$).
- Cut at 4 kHz passes 1 kHz and 3 kHz, stops 5 kHz: a rounded square with visible ripple at the flat tops.
- The harmonic amplitudes fall as $1/k$, so any cut leaves a usable but imperfect waveform – bandwidth is chosen by the distortion you accept.

3.2

- USB 100–104 kHz, LSB 96–100 kHz.
- Total DSB bandwidth 8 kHz.
- SSB (USB): 100–104 kHz, 4 kHz total.
- The 8 kHz channel fits DSB exactly; SSB would also fit and leave room, but it costs receiver complexity (coherent detection, no simple envelope detector) and the transmitter's carrier-suppression accuracy.