

10.7 PID control and compensation

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS PID controller PID • lead compensator

Objective

Tune PID terms against measured specs, size lead compensators in the frequency domain, and handle saturation with anti-windup.

1 Key knowledge and worked examples

■ The three terms

$C = K_p + K_i/s + K_d s$: P reacts to present error, I to its history (kills steady-state error, adds a type), D to its slope (adds phase, dies in noise). Saturation invalidates the linear story – the integrator winds up unless clamped or back-calculated.

■ Worked example: Ziegler–Nichols from the edge

Plant $1/[s(s+1)(s+2)]$: Routh gives $K_u = 6$ (margin at $\omega_u = \sqrt{2} = 1.41$), so $T_u = 2\pi/1.41 = 4.44s$. ZN PID: $K_p = 0.6K_u = 3.6$, $T_i = T_u/2 = 2.22s$, $T_d = T_u/8 = 0.555s$ – and the closed-loop characteristic computes to $s^4 + 3s^3 + 4.0s^2 + 3.6s + 1.62$ with all four roots in the LHP ✓.

■ Lead sizing

$(1+\tau s)/(1+\alpha\tau s)$, $0 < \alpha < 1$: max phase $\sin\varphi_m = (1-\alpha)/(1+\alpha)$ at $\omega_m = 1/(\tau\sqrt{\alpha})$. For $+40^\circ$: $\alpha = 0.217$; high-frequency gain rises by $1/\alpha = 4.6$ – the price the rest of the design pays.

2 Practice

2.1 A plant oscillates at $K_u = 8$, $T_u = 1.5s$. Give the ZN settings for P and for PI. [3]

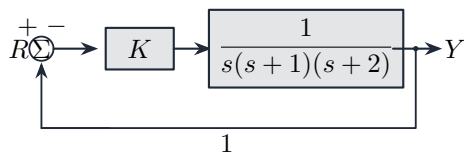
2.2 Compute α for a lead network aiming at $+40^\circ$ maximum boost. [3]

2.3 A P-only loop $K_p = 5$ around $4/[(s+1)(s+2)]$: compute the unit-step steady-state error, and state what adding integral action does to it. [3]

3 Exam-style questions

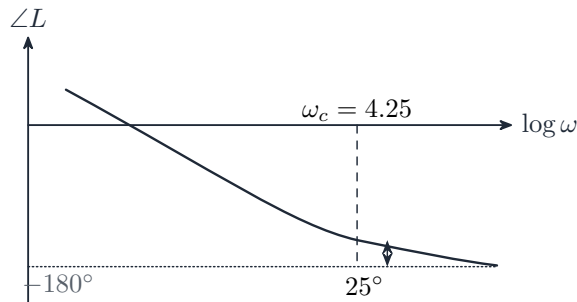
3.1 A unity-feedback loop around the plant $G = 1/[s(s+1)(s+2)]$ is to be tuned by Ziegler–Nichols.

Show from Routh that proportional gain first loses stability at $K_u = 6$, and give the oscillation frequency. Compute T_u and the full Ziegler–Nichols PID settings. Explain what each of the three terms contributes here (error type, phase effect), and why ZN settings are a starting point rather than a final tuning. [8]



3.2 A loop $L = 20/[s(s + 2)]$ has $\text{PM} = 25^\circ$ ($\omega_c = 4.25 \text{ rad/s}$) and must reach at least 45° .

Compute the phase boost needed at the new crossover and add margin for the crossover shift (design the lead for $+30^\circ$). Size α and state the gain the network adds at high frequency. Choose τ so the boost lands at $\omega_m \approx 4.5 \text{ rad/s}$, giving the compensator's zero and pole. State the remaining step of the design (re-locate the crossover) and why. [7]



Solutions

2.1

- P: $K_p = 0.5K_u = 4$.
- PI: $K_p = 0.45K_u = 3.6$, $T_i = T_u/1.2 = 1.25s$.

2.2

- $\sin 40^\circ = 0.643 = (1 - \alpha)/(1 + \alpha) \rightarrow \alpha = 0.37/1.64 = 0.217$.

2.3

- Loop gain at dc = $5 \times 4/2 = 10 \rightarrow e_{ss} = 1/(1 + 10) = 0.0909$.
- Integral action raises the system type: the error goes to exactly zero (with the phase-lag cost at crossover).

3.1

- Characteristic $s^3 + 3s^2 + 2s + K$: s^1 row $(6 - K)/3 \rightarrow$ instability at $K_u = 6$.
- At the boundary the auxiliary $3s^2 + 6 = 0 \rightarrow \omega_u = \sqrt{2} = 1.41rad/s$.
- $T_u = 2\pi/1.41 = 4.44s$.
- PID: $K_p = 3.6$, $T_i = 2.22s$, $T_d = 0.555s$.
- Contributions: I raises the type to 2 (zero ramp error) but adds 90° of phase lag at low frequency; D contributes phase lead around crossover, restoring damping; P supplies the loop gain the other two shape.
- ZN tunes a quarter-decay response on the stability edge – aggressive and plant-specific: always re-tune against the real spec (overshoot, noise, saturation), especially since D amplifies measurement noise.

3.2

- Deficit: $45 - 25 = 20^\circ$; designing for $+30^\circ$ covers the crossover shift.
- $\alpha = (1 - \sin 30^\circ)/(1 + \sin 30^\circ) = 0.5/1.5 = 0.333$.
- High-frequency gain $1/\alpha = 3$.
- $\omega_m = 1/(\tau\sqrt{\alpha}) = 4.5 \rightarrow \tau = 1/(4.5 \times 0.577) = 0.385s$.
- Zero at $1/\tau = 2.60rad/s$, pole at $1/(\alpha\tau) = 7.79rad/s$.
- Remaining: the lead lifts the gain near ω_m by $1/\sqrt{\alpha} = 1.73$, so the 0-dB crossing moves right – re-evaluate the new ω_c and confirm the -180° distance (iterate once if the boost landed off the new crossover).