

10.6 Bode and Nyquist analysis

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS gain margin • phase margin • Nyquist criterion

Objective

Draw Bode plots, read gain and phase margins off them, apply the Nyquist criterion with correct encirclement counting, and connect margins to closed-loop ringing.

1 Key knowledge and worked examples

■ Margins

Gain margin: how many times the gain can rise before $|L| = 1$ at the -180° crossing. Phase margin: $PM = 180^\circ + \angle L(j\omega_c)$ at the gain crossover. Working rule: $\zeta \approx PM/100$ – PM $50^\circ \approx 16\%$ overshoot.

■ Worked example: PM by formula

$L = 25/[s(s+5)]$: crossover solves $\omega\sqrt{\omega^2+25} = 25 \rightarrow \omega_c = 3.93$; phase $-90^\circ - \text{atan}(3.93/5) = -128.2^\circ \rightarrow PM = 51.8^\circ$ – a well-damped design by two lines of arithmetic.

■ Nyquist in one number

$L = K/[s(s+2)(s+4)]$ crosses -180° at $\omega = \sqrt{8} = 2.83$ where $|L| = K/48$: the -1 encirclement (and stability) flips at $K = 48$ – exactly Routh's answer for $s^3 + 6s^2 + 8s + K$.

2 Practice

2.1 Evaluate magnitude (dB) and phase of $10/(s+1)$ at $\omega = 1$ rad/s. [3]

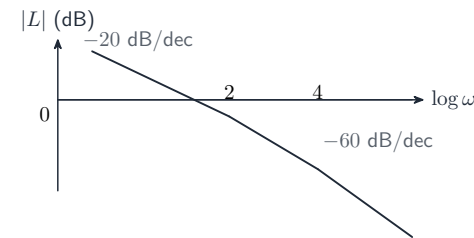
2.2 Compute PM for $L = 25/[s(s+5)]$. [3]

2.3 For $K = 12$ in $L = K/[s(s+2)(s+4)]$, compute GM (as a factor and in dB). [3]

3 Exam-style questions

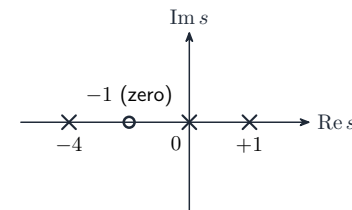
3.1 A loop $L = 24/[s(s+2)(s+4)]$ (unity feedback).

Compute the gain crossover frequency (solve $\omega\sqrt{(\omega^2+4)(\omega^2+16)} = 24$). Compute the phase margin there. Compute GM from the -180° crossing. Predict ζ and the overshoot from the PM using the rule, and check the trend against the Routh stability bound ($K < 48$). [8]



3.2 A plant has open loop $L = K(s+1)/[s(s-1)(s+4)]$ – note the pole at $+1$.

State P (open-loop RHP poles). Write the characteristic polynomial and use Routh to find the range of K for closed-loop stability. State what the Nyquist criterion then REQUIRES the plot to do (how many encirclements, which way) for stability in that range, and what it does for K just below the boundary. [7]



Solutions

2.1

- $|L| = 10/\sqrt{2} = 7.07 \rightarrow 20\log_{10}7.07 = 17.0\text{dB}$.
- Phase $= -\text{atan}(1) = -45^\circ$.

2.2

- $\omega_c = 3.93\text{rad/s}$.
- Phase $-90^\circ - \text{atan}(3.93/5) = -128.2^\circ$.
- $PM = 180 - 128.2 = 51.8^\circ$.

2.3

- At $\omega = 2.83$: $|L| = 12/48 = 0.25$.
- $GM = 1/0.25 = 4$ (12 dB).

3.1

- Solve $\omega\sqrt{(\omega^2 + 4)(\omega^2 + 16)} = 24$: $\omega_c = 1.94$ rad/s.
- Phase $= -90 - \text{atan}(1.94/2) - \text{atan}(1.94/4) = -160.0^\circ$.
- $PM = 180 - 160 = 20^\circ$.
- At the -180° crossing ($\omega = 2.83$): $|L| = 24/48 = 0.5 \rightarrow GM = 2 = 6$ dB.
- Rule: $\zeta \approx 20/100 = 0.2 \rightarrow$ overshoot near $e^{-\pi \cdot 0.2/0.98} = 53\%$ – ringy, and only a factor 2 in gain from instability ($GM = 2$, boundary $K = 48$).
- Trend check: the two-pole loop $25/[s(s+5)]$ enjoys PM 51.8° ; adding the third pole (at -4 , near crossover) drops PM to 20° at comparable gain – each pole near crossover donates phase lag and eats margin.

3.2

- $P = 1$ (the pole at $s = +1$).
- Characteristic: $s(s-1)(s+4) + K(s+1) = s^3 + 3s^2 + (K-4)s + K$.
- Routh: s^3 : 1, $K-4$; s^2 : 3, K ; s^1 : $(3(K-4) - K)/3 = (2K-12)/3$.
- Stable for $K > 6$ (both $K > 4$ and $K > 6$ reduce to $K > 6$).
- Nyquist for stability: $Z = N + P = 0 \rightarrow N = -1$ – exactly ONE counter-clockwise encirclement of -1 required.
- For K just below 6 the encirclement disappears ($N = 0 \rightarrow Z = 1$): one closed-loop pole in the RHP – the plot no longer wraps -1 .