

10.4 Routh stability

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS Routh array

Objective

Build Routh arrays, count right-half-plane roots without solving, find stability ranges for a parameter, and handle the special cases correctly.

1 Key knowledge and worked examples

■ The array

Two rows from odd/even coefficients, each new row by cross-multiplication; sign changes in the first column = RHP roots. A zero first-column entry needs epsilon; a whole zero row needs the auxiliary equation.

■ Worked example: two sign changes

$s^4 + 2s^3 + 3s^2 + 4s + 5$: rows give first column 1, 2, 1, -6, 5 – two sign changes, two RHP roots (exact roots: $-1.29 \pm j0.86$ and $+0.29 \pm j1.42$ ✓).

■ Worked example: the zero row

$s^5 + 2s^4 + 24s^3 + 48s^2 + 50s + 100$ produces a zero s^3 row: the auxiliary equation $2s^4 + 48s^2 + 100 = 0 \rightarrow j\omega$ roots $\pm j1.52$, $\pm j4.66$ and one LHP root at -2 – marginally stable (sustained oscillation), not asymptotically stable.

2 Practice

2.1 Build the Routh array for $s^3 + 4s^2 + 8s + 12$ and state whether the system is stable. [3]

2.2 How many RHP roots has $s^4 + s^3 + 2s^2 + 4s + 3$? [3]

2.3 For $s^3 + 3s^2 + 3s + (1 + K)$, find the range of K for stability. [3]

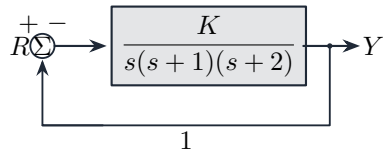
3 Exam-style questions

3.1 The polynomial $s^5 + 2s^4 + 24s^3 + 48s^2 + 50s + 100$ arises as a closed-loop characteristic equation.

Build the Routh array as far as the s^3 row and show that it is a zero row. Form the auxiliary equation from the s^4 row and solve it. Complete the array using the derivative row and state the stability conclusion (asymptotically stable / marginally stable / unstable), justifying from the array. [8]

3.2 A unity-feedback loop has plant $K/[s(s+1)(s+2)]$.

Write the characteristic equation. Apply Routh to find the stability range of K. At the critical K, give the frequency of sustained oscillation (via the auxiliary equation). Confirm by computing the roots at $K = 4$ that the system is inside the stable range there. [7]



Solutions

2.1

- s^3 : 1, 8; s^2 : 4, 12; s^1 : $(32-12)/4 = 5$; s^0 : 12.
- First column 1, 4, 5, 12 all positive $\rightarrow$ stable.

2.2

- s^4 : 1, 2, 3; s^3 : 1, 4; s^2 : $(2-4) = -2$, 3; s^1 : large positive; s^0 : 3.
- First column 1, 1, -2 , ..., 3: two sign changes $\rightarrow$ 2 RHP roots (exact: $+0.46 \pm j1.55$, $-0.96 \pm j0.47$ ✓).

2.3

- s^3 : 1, 3; s^2 : 3, $1+K$; s^1 : $(9 - (1+K))/3 = (8-K)/3$.
- Stable for $-1 < K < 8$.

3.1

- s^5 : 1, 24, 50; s^4 : 2, 48, 100.
- s^3 row: $((2 \times 24 - 48)/2, (2 \times 50 - 100)/2) = (0, 0)$ – a full zero row.
- Auxiliary equation from s^4 : $2s^4 + 48s^2 + 100 = 0$; substitute $u = s^2$: $u = (-48 \pm \sqrt{2304 - 800})/4$ gives $s^2 = -2.30$ or -21.7 , so $s = \pm j1.52, \pm j4.66$.
- Replace the zero row with d/ds of the auxiliary: $8s^3 + 96s \rightarrow 8, 96$ (s^3 row), continue: all remaining first-column entries positive.
- Zero RHP roots BUT a $j\omega$ pair $\rightarrow$ marginally stable: bounded sustained oscillation, poles neither decay nor grow.

3.2

- Characteristic: $s^3 + 3s^2 + 2s + K = 0$.
- s^3 : 1, 2; s^2 : 3, K ; s^1 : $(6 - K)/3$.
- Stable for $0 < K < 6$.
- At $K = 6$ the s^1 row is zero; auxiliary from s^2 : $3s^2 + 6 = 0 \rightarrow s = \pm j\sqrt{2}$ – oscillation at $\omega = \sqrt{2} = 1.414 \text{ rad/s}$.
- At $K = 4$: $0 < 4 < 6 \rightarrow$ inside the stable range; roots $-2.80, -0.10 \pm j1.19$ (all LHP ✓).