

10.2 Transfer functions and block diagrams

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS transfer function • block diagram

Objective

Reduce interconnected blocks to one transfer function: series, parallel and loop algebra, with Mason's rule for genuinely tangled diagrams.

1 Key knowledge and worked examples

■ The three moves

Series multiplies: $G_1 G_2$. Parallel adds: $G_1 + G_2$. Feedback: $T = G/(1 + GH)$ – always reduce inner loops first.

■ Worked example: two loops in layers

Inner loop $2/s$ with feedback $0.5 \rightarrow 2/(s+1)$; series $1/(s+1)$; outer unity feedback $\rightarrow T = 2/[(s+1)^2 + 2] = 2/(s^2 + 2s + 3) - \omega_n = 1.73, \zeta = 0.577 \checkmark$ the damping falls because the pole pair moved from the real axis.

■ Non-unity feedback

A sensor in the loop is real feedback: with $G = 10/[s(s+2)]$, $H = 0.2$: $T = G/(1 + GH) = 10/(s^2 + 2s + 2) - \omega_n = 1.41, \zeta = 0.707$. The denominator is what matters; the numerator rarely changes shape.

2 Practice

2.1 Reduce the series chain $2/(s+1) \times 1/(s+3)$ to one transfer function. [3]

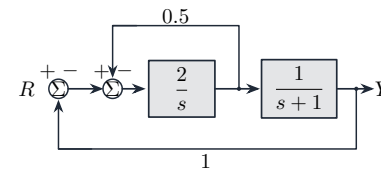
2.2 Reduce the parallel pair $1/s$ and $1/(s+2)$ to one block. [3]

2.3 Write the closed-loop transfer function for $G = 4/(s+2)$ under unity feedback, and its dc gain. [3]

3 Exam-style questions

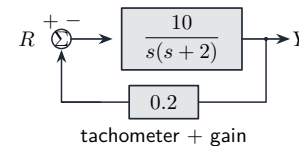
3.1 A system has an inner loop: $G_1 = 2/s$ with feedback $H_1 = 0.5$, in series with $G_2 = 1/(s+1)$, all closed by an outer unity-feedback loop.

Reduce the inner loop. Reduce the series chain. Close the outer loop and give $T(s)$ with a monic denominator. Read off ω_n and ζ and classify the response. State the order of the final system and why the inner-loop reduction did not change it. [8]



3.2 A position servo: plant $G = 10/[s(s+2)]$, sensor (tachometer + gain) $H = 0.2$.

Write the closed-loop transfer function $T = G/(1 + GH)$. Give the characteristic polynomial, ω_n and ζ . Compute the closed-loop dc gain and explain why it is not 1 even though the open-loop plant has an integrator. [7]



Solutions

2.1

- $2/[(s+1)(s+3)]$.

2.2

- $(2s+2)/[s(s+2)]$ – common denominator, add numerators.

2.3

- $T = 4/(s+2)/(1+4/(s+2)) = 4/(s+6)$.
- Dc gain $4/6 = 0.667$.

3.1

- Inner: $(2/s)/(1+2/s \times 0.5) = 2/(s+1)$.
- Series: $2/[(s+1)(s+1)]$.
- Outer: $T = 2/[(s+1)^2+2] = 2/(s^2+2s+3)$.
- $\omega_n = \sqrt{3} = 1.73rad/s$, $\zeta = 2/(2\sqrt{3}) = 0.577$.
- Underdamped ($\zeta < 1$): overshoot about 11% before settling.
- Second order throughout: the inner loop collapses to one first-order block, and one such block in a unity loop is second order – reduction changes algebra, not order.

3.2

- $GH = 10 \times 0.2/[s(s+2)] = 2/[s(s+2)]$; $T = 10/[s(s+2)+2] = 10/(s^2+2s+2)$.
- $\omega_n = \sqrt{2} = 1.414rad/s$; $\zeta = 2/(2\sqrt{2}) = 0.707$.
- Dc gain $10/2 = 5$.
- The integrator sits in the forward path, but the FEEDBACK is not unity: the error is not the input minus output, so the integrator no longer forces a perfect match – with unity feedback the dc gain would be exactly 1.