

# 10.2 Transfer functions and block diagrams

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 24 marks

KEY WORDS transfer function • block diagram

## Objective

Reduce interconnected blocks to one transfer function: series, parallel and loop algebra, with Mason's rule for genuinely tangled diagrams.

## 1 Key knowledge and worked examples

### ■ The three moves

Series multiplies:  $G_1 G_2$ . Parallel adds:  $G_1 + G_2$ . Feedback:  $T = G/(1 + GH)$  – always reduce inner loops first.

### ■ Worked example: two loops in layers

Inner loop  $2/s$  with feedback  $0.5 \rightarrow 2/(s+1)$ ; series  $1/(s+1)$ ; outer unity feedback  $\rightarrow T = 2/[(s+1)^2 + 2] = 2/(s^2 + 2s + 3) - \omega_n = 1.73, \zeta = 0.577 \checkmark$  the damping falls because the pole pair moved from the real axis.

### ■ Non-unity feedback

A sensor in the loop is real feedback: with  $G = 10/[s(s+2)]$ ,  $H = 0.2$ :  $T = G/(1 + GH) = 10/(s^2 + 2s + 2) - \omega_n = 1.41, \zeta = 0.707$ . The denominator is what matters; the numerator rarely changes shape.

## 2 Practice

2.1 Reduce the series chain  $2/(s+1) \times 1/(s+3)$  to one transfer function. [3]

2.2 Reduce the parallel pair  $1/s$  and  $1/(s+2)$  to one block. [3]

**2.3** Write the closed-loop transfer function for  $G = 4/(s + 2)$  under unity feedback, and its dc gain. [3]

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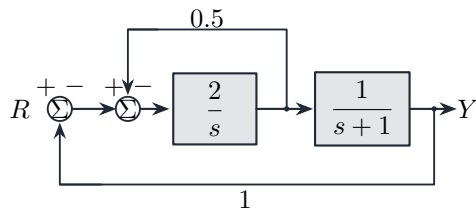
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### 3 Exam-style questions

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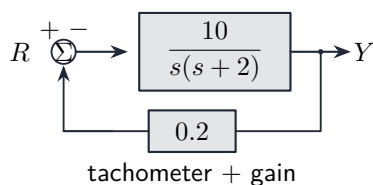
**3.1** A system has an inner loop:  $G_1 = 2/s$  with feedback  $H_1 = 0.5$ , in series with  $G_2 = 1/(s + 1)$ , all closed by an outer unity-feedback loop.

Reduce the inner loop. Reduce the series chain. Close the outer loop and give  $T(s)$  with a monic denominator. Read off  $\omega_n$  and  $\zeta$  and classify the response. State the order of the final system and why the inner-loop reduction did not change it. [8]



**3.2** A position servo: plant  $G = 10/[s(s + 2)]$ , sensor (tachometer + gain)  $H = 0.2$ .

Write the closed-loop transfer function  $T = G/(1 + GH)$ . Give the characteristic polynomial,  $\omega_n$  and  $\zeta$ . Compute the closed-loop dc gain and explain why it is not 1 even though the open-loop plant has an integrator. [7]



## Solutions

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### 2.1

- $2/[(s+1)(s+3)]$ .

### 2.2

- $(2s+2)/[s(s+2)]$  – common denominator, add numerators.

### 2.3

- $T = 4/(s+2)/(1+4/(s+2)) = 4/(s+6)$ .
- Dc gain  $4/6 = 0.667$ .

### 3.1

- Inner:  $(2/s)/(1+2/s \times 0.5) = 2/(s+1)$ .
- Series:  $2/[(s+1)(s+1)]$ .
- Outer:  $T = 2/[(s+1)^2+2] = 2/(s^2+2s+3)$ .
- $\omega_n = \sqrt{3} = 1.73\text{rad/s}$ ,  $\zeta = 2/(2\sqrt{3}) = 0.577$ .
- Underdamped ( $\zeta < 1$ ): overshoot about 11% before settling.
- Second order throughout: the inner loop collapses to one first-order block, and one such block in a unity loop is second order – reduction changes algebra, not order.

### 3.2

- $GH = 10 \times 0.2/[s(s+2)] = 2/[s(s+2)]$ ;  $T = 10/[s(s+2)+2] = 10/(s^2+2s+2)$ .
- $\omega_n = \sqrt{2} = 1.414\text{rad/s}$ ;  $\zeta = 2/(2\sqrt{2}) = 0.707$ .
- Dc gain  $10/2 = 5$ .
- The integrator sits in the forward path, but the FEEDBACK is not unity: the error is not the input minus output, so the integrator no longer forces a perfect match – with unity feedback the dc gain would be exactly 1.