

1.7 First-order RC and RL response

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS natural response • step response • time constant
• flyback diode

Objective

Write first-order RC and RL responses directly from initial value, final value and time constant; solve switching problems; and explain – with numbers – the hazards of interrupting inductor current.

1 Key knowledge and worked examples

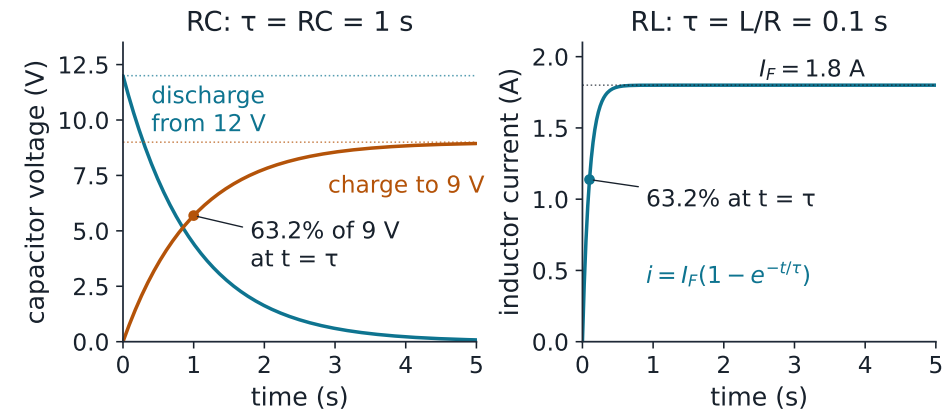
■ One formula for every first-order circuit

Every circuit with one storage element answers to

$$v(t) = V_F + (V_0 - V_F)e^{-t/\tau}$$

V_0 comes from the stored energy before the switch (voltage continuity for C, current continuity for L); V_F comes from the circuit after the switch, with the element replaced by its steady-state open (C) or short (L); $\tau = RC$ or $\tau = L/R$ uses the resistance the element sees with sources zeroed.

At $t = \tau$ the gap has closed to 37%; after 5τ the circuit is within 1% and treated as settled.



■ Worked example: discharge and charge with the same τ

A 100 μF capacitor charged to 12 V discharges through 10 k Ω : $\tau = 1\text{s}$, so $v(t) = 12e^{-t}$ V. After 2 s: $12e^{-2} \approx 1.62\text{V}$.

Now switch the same RC to a 9 V source from 0 V: $V_0 = 0$, $V_F = 9$, so $v(t) = 9(1 - e^{-t})$; at $t = 1\text{s}$ it reaches $9 \times 0.632 \approx 5.7\text{V}$. The exponent's argument is dimensionless because τ carries the seconds.

■ Worked example: RL rise and the switch-off spike

$R = 8\Omega$, $L = 0.4\text{H}$ across a 12 V source: $I_F = 1.5\text{A}$, $\tau = L/R = 50\text{ms}$, $i(t) = 1.5(1 - e^{-t/0.05})$ A; at 100 ms, $1.5 \times 0.865 \approx 1.30\text{A}$.

Opening the switch forces the 1.5 A to stop in, say, 1 ms through a 0.1 H winding: $v = L di/dt = 0.1 \times 1500 = 150\text{V}$ – tens of times the supply, and the arc across the opening contacts is that voltage made visible. A **flyback diode** gives the current a path so di/dt stays small.

2 Practice

2.1 Compute the time constant of 22 k Ω with 470 nF, and the settling time to within 1%. [3]

2.2 An RL circuit has $R = 8\Omega$, $L = 0.4H$ and a 12 V source. Give I_F , τ , and the current at $t = 0.1s$ starting from zero. [4]

2.3 A 5 V source, 1 k Ω and 100 μF start with the capacitor at 1 V. Give V_0 , V_F , τ and v at $t = 0.1s$. [4]

3 Exam-style questions

3.1 A 9 V source charges 100 μF through 10 k Ω until fully charged; the capacitor is then switched to discharge through 20 k Ω .

Write both responses $v(t)$ with their time constants on the same time axis (reset $t = 0$ at the switch). Then compute the time after the switch for the voltage to fall below 0.5 V, and verify the fully-charged claim using the 5τ rule for the charging phase. [7]

3.2 A relay coil models as 0.1 H with 20 Ω of resistance, driven from 12 V. The driver transistor interrupts the 0.6 A coil current in about 1 ms when switching off.

Compute the steady current, the coil's time constant, and the voltage the coil generates across the opening transistor if nothing limits it. Explain how a flyback diode changes this, and give the diode's required reverse rating. [6]

Solutions

2.1

- $\tau = RC = 22000 \times 470 \times 10^{-9} \approx 10.3ms$.
- Within 1% at $5\tau \approx 51.7ms$.
- (Roughly 52 ms is the standard settling estimate.)

2.2

- $I_F = 12/8 = 1.5A$.
- $\tau = L/R = 0.4/8 = 50ms$.
- $i(0.1) = 1.5(1 - e^{-2}) = 1.5 \times 0.865 \approx 1.30A$.

2.3

- $V_0 = 1V$ (continuity), $V_F = 5V$, $\tau = 0.1s$.
- $v(0.1) = 5 - 4e^{-1} = 5 - 1.47 = 3.53V$.

3.1

- Charging: $v = 9(1 - e^{-t})$ with $\tau = 1s$.
- Charging needs $5\tau = 5s$ to be within 1% of 9 V – ”fully charged” is fair.
- After the switch, $\tau = 20k \times 100\mu = 2s$ and $v = 9e^{-t/2}$.
- Fall below 0.5 V: $9e^{-t/2} < 0.5 \rightarrow t > 2\ln(18) \approx 5.78s$.
- Check endpoints: at $t = 0$ the discharge starts at 9 V ✓; it never goes below zero ✓.

3.2

- Steady current: $I = 12/20 = 0.6A$ (inductor is a short at DC – matches the given interrupting current).
- $\tau = L/R = 0.1/20 = 5ms$.
- Unlimited interruption: $v = Ldi/dt = 0.1 \times (0.6/0.001) = 60V$.
- A flyback diode across the coil (cathode to +12 V) gives the current a loop to keep circulating; di/dt is set by the coil resistance, so the spike collapses to about a 0.7 V diode drop plus IR.
- The diode must withstand 12 V in reverse when the driver is on.