

1.6 Capacitors, inductors and energy

Name: _____ Class: _____ Date: _____ Total: 22 marks

KEY WORDS capacitor • capacitance • inductor • inductance
• energy

Objective

Relate capacitor and inductor behaviour to their defining laws, combine them correctly, compute stored energy, and explain why their stored quantities cannot jump.

1 Key knowledge and worked examples

■ Two storage elements, two mirror-image laws

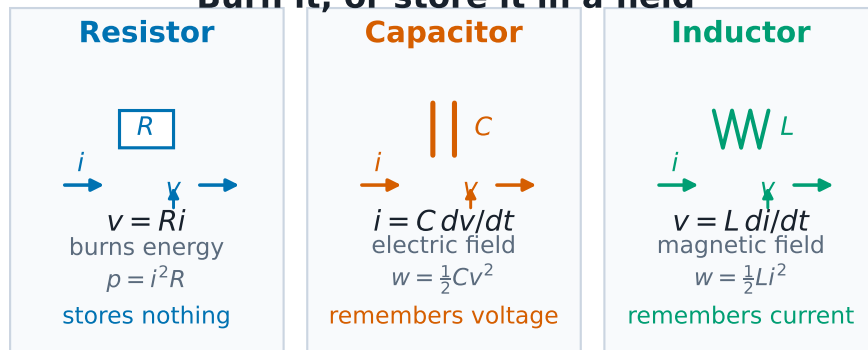
A **capacitor** stores separated charge; its **capacitance** C (farads) gives $q = Cv$ and

$$i = C \frac{dv}{dt}$$

An **inductor** stores energy in a magnetic field; its **inductance** L (henrys) gives $v = L \frac{di}{dt}$. Steady DC makes the capacitor an open circuit and the inductor a short circuit – that single rule solves most steady-state problems.

Stored energy depends only on the present state: $w_C = \frac{1}{2}Cv^2$, $w_L = \frac{1}{2}Li^2$.

Burn it, or store it in a field



■ Worked example: combinations are mirror images

Capacitances in **parallel** add (areas add): $47\mu F + 47\mu F = 94\mu F$. In **series** the reciprocal rule applies: $47/2 = 23.5 \mu F$. Inductances behave the opposite way: series adds, parallel divides – the exact mirror of resistors.

A $100 \mu F$ capacitor charged to 12 V holds $w = \frac{1}{2} \times 100 \times 10^{-6} \times 144 = 7.2 mJ$ – enough to reset electronics, which is why large capacitors get discharged before you touch them.

■ Worked example: rates, and the continuity argument

With $C = 220\mu F$ and the voltage ramping $0 \rightarrow 10$ V in 50 ms: $i = Cdv/dt = 220 \times 10^{-6} \times 200 = 44 mA$ while the ramp lasts.

An inductor $L = 20 mH$ carrying 3 A holds $w = \frac{1}{2} \times 0.02 \times 9 = 90 mJ$. If its current is forced to change at 200 A/s, the terminal voltage is $v = Ldi/dt = 4V$. And because w_L cannot jump, **inductor current is continuous** – forcing it to zero instantly demands an enormous di/dt , and the resulting voltage arcs across the opening contact. Ignition coils work on exactly this principle.

2 Practice

2.1 Two $47 \mu F$ capacitors: (a) in parallel, (b) in series. Give both equivalent capacitances, then state which connection stores more total energy at 10 V. [4]

2.2 A 20 mH inductor carries 3 A. Compute the stored energy. Then, if the current falls at 200 A/s, give the inductor's terminal voltage and its polarity relative to the current direction. [4]

2.3 A 220 μF capacitor's voltage ramps from 0 V to 10 V in 50 ms and then holds steady. Give the current during the ramp and after it, with one sentence explaining the after value. [3]

3 Exam-style questions

3.1 A 470 μF capacitor is charged to 20 V and then discharged through a resistor. Compute the stored energy. Explain, in terms of the defining law $i = C \, dv/dt$, why the capacitor voltage cannot jump to zero in zero time, and state where the energy goes. If the discharge takes 2 s at roughly constant average power, estimate that power. [6]

3.2 A 12 V source feeds 100 Ω in series; node A then has a 200 Ω resistor in parallel with a large capacitor. Using DC steady-state reasoning, find the current in the 100 Ω , the voltage across the capacitor, and the energy it stores (take $C = 100\mu\text{F}$). Then state, with a reason, what the capacitor's current is at steady state. [5]

Solutions

2.1

- Parallel: $47 + 47 = 94 \text{ } \mu\text{F}$.
- Series: $47 \times 47/94 = 23.5 \mu\text{F}$.
- Energy at 10 V: parallel $\frac{1}{2} \times 94 \mu \times 100 = 4.7 \text{mJ}$; series $\frac{1}{2} \times 23.5 \mu \times 100 = 1.18 \text{mJ}$.
- Parallel stores more (larger C at the same voltage).

2.2

- Stored energy: $\frac{1}{2} \times 0.02 \times 3^2 = 90 \text{mJ}$.
- Terminal voltage: $0.02 \times 200 = 4 \text{V}$.
- With falling current, the induced voltage drives current onward: the terminal that current enters behaves as $+$ – the inductor momentarily acts as a source.

2.3

- During the ramp: $i = 220 \mu \times (10/0.05) = 44 \text{mA}$.
- After: $i = 0$.
- Constant voltage means $dv/dt = 0$, and $i = C dv/dt = 0$ – an open circuit to steady DC.

3.1

- Stored energy: $\frac{1}{2} \times 470 \times 10^{-6} \times 400 = 94 \text{mJ}$.
- A voltage jump would need $dv/dt \rightarrow \infty$, hence infinite $i = C dv/dt$ – impossible with any real (finite-current) path.
- The energy is dissipated as heat in the resistor (some in the capacitor's own resistance).
- Average power $\approx 0.094 \text{J}/2\text{s} \approx 47 \text{mW}$.
- Consistency: initial power is V^2/R , far larger; the estimate is an average, which is what was asked.

3.2

- At DC steady state the capacitor is an open circuit, so it carries no current.
- The 200Ω stands alone in the loop: $I = 12/300 = 40 \text{mA}$.
- Capacitor voltage equals the 200Ω voltage: $12 \times 200/300 = 8 \text{V}$.
- Energy: $\frac{1}{2} \times 100 \times 10^{-6} \times 64 = 3.2 \text{mJ}$; capacitor current at steady state: zero.