1.6 Capacitors, inductors and energy

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS capacitor • capacitance • inductor • inductance • energy

Objective

Relate capacitor and inductor behaviour to their defining laws, combine them correctly, compute stored energy, and explain why their stored quantities cannot jump.

1 Key knowledge and worked examples

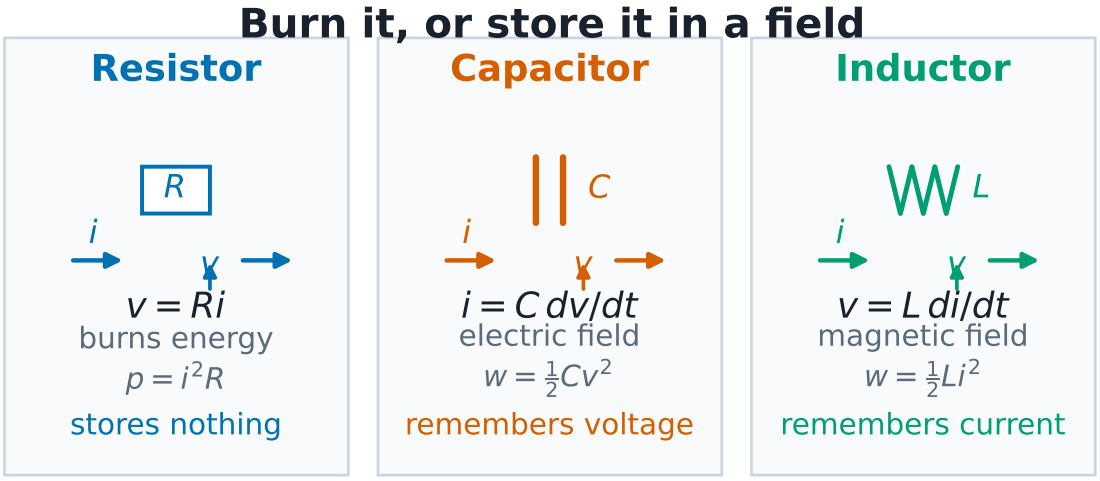
■ **Two storage elements, two mirror-image laws**

A **capacitor** stores separated charge; its **capacitance** `C` (farads) gives `q = Cv` and

$$i = C\,\frac{dv}{dt}$$

An **inductor** stores energy in a magnetic field; its **inductance** `L` (henrys) gives `v = L\,di/dt`. Steady DC makes the capacitor an open circuit and the inductor a short circuit – that single rule solves most steady-state problems.

Stored energy depends only on the present state: $w_C = \tfrac{1}{2}Cv^2$, $w_L = \tfrac{1}{2}Li^2$.



■ Worked example: combinations are mirror images

Capacitances in **parallel** add (areas add): $47\mu F + 47\mu F = 94\mu F$. In **series** the reciprocal rule applies: `47/2 = 23.5 µF`. Inductances behave the opposite way: series adds, parallel divides – the exact mirror of resistors.

A 100 µF capacitor charged to 12 V holds $w = ½ \times 100 \times 10^{-6} \times 144 = 7.2mJ$ – enough to reset electronics, which is why large capacitors get discharged before you touch them.

■ Worked example: rates, and the continuity argument

With $C = 220\mu F$ and the voltage ramping $0 \to 10$ V in 50 ms: $i = Cdv/dt = 220 \times 10^{-6} \times 200 = 44mA$ while the ramp lasts.

An inductor $L = 20mH$ carrying 3 A holds $w = ½ \times 0.02 \times 9 = 90mJ$. If its current is forced to change at 200 A/s, the terminal voltage is $v = Ldi/dt = 4V$. And because w_L cannot jump, **inductor current is continuous** – forcing it to zero instantly demands an enormous `di/dt`, and the resulting voltage arcs across the opening contact. Ignition coils work on exactly this principle.

2 Practice

2.1 Two 47 µF capacitors: (a) in parallel, (b) in series. Give both equivalent capacitances, then state which connection stores more total energy at 10 V. [4]

2.2 A 20 mH inductor carries 3 A. Compute the stored energy. Then, if the current falls at 200 A/s, give the inductor's terminal voltage and its polarity relative to the current direction. [4]

2.3 A 220 µF capacitor's voltage ramps from 0 V to 10 V in 50 ms and then holds steady. Give the current during the ramp and after it, with one sentence explaining the after value. [3]

3 Exam-style questions

3.1 A 470 µF capacitor is charged to 20 V and then discharged through a resistor.

Compute the stored energy. Explain, in terms of the defining law `i = C dv/dt`, why the capacitor voltage cannot jump to zero in zero time, and state where the energy goes. If the discharge takes 2 s at roughly constant average power, estimate that power. [6]

3.2 A 12 V source feeds 100 Ω in series; node A then has a 200 Ω resistor in parallel with a large capacitor.

Using DC steady-state reasoning, find the current in the 100 Ω, the voltage across the capacitor, and the energy it stores (take $C = 100\mu F$). Then state, with a reason, what the capacitor's current is at steady state. [5]

Solutions

2.1

- Parallel: `47 + 47 = 94 µF`.
- Series: $47 \times 47/94 = 23.5\mu F$.
- Energy at 10 V: parallel $\frac{1}{2} \times 94\mu \times 100 = 4.7mJ$; series $\frac{1}{2} \times 23.5\mu \times 100 = 1.18mJ$.
- Parallel stores more (larger C at the same voltage).

2.2

- Stored energy: $\frac{1}{2} \times 0.02 \times 3^2 = 90mJ$.
- Terminal voltage: $0.02 \times 200 = 4V$.
- With falling current, the induced voltage drives current onward: the terminal that current enters behaves as `+` – the inductor momentarily acts as a source.

2.3

- During the ramp: $i = 220\mu \times (10/0.05) = 44mA$.
- After: $i = 0$.
- Constant voltage means $dv/dt = 0$, and $i = Cdv/dt = 0$ – an open circuit to steady DC.

3.1

- Stored energy: $\frac{1}{2} \times 470 \times 10^{-6} \times 400 = 94mJ$.
- A voltage jump would need $dv/dt \to \infty$, hence infinite `i = C dv/dt` – impossible with any real (finite-current) path.
- The energy is dissipated as heat in the resistor (some in the capacitor's own resistance).
- Average power $\approx 0.094J/2s \approx 47mW$.
- Consistency: initial power is V^2/R, far larger; the estimate is an average, which is what was asked.

3.2

- At DC steady state the capacitor is an open circuit, so it carries no current.
- The 200 Ω stands alone in the loop: $I = 12/300 = 40mA$.
- Capacitor voltage equals the 200 Ω voltage: $12 \times 200/300 = 8V$.
- Energy: $\frac{1}{2} \times 100 \times 10^{-6} \times 64 = 3.2mJ$; capacitor current at steady state: zero.