

1.5 Ideal operational amplifiers

Name: _____ Class: _____ Date: _____ Total: 21 marks

KEY WORDS operational amplifier
 • negative feedback • virtual short • ideal operational amplifier
 • saturation • inverting amplifier

Objective

Analyse ideal op-amp circuits with negative feedback using the two golden rules, compute inverting and non-inverting gains, and recognise when saturation ends the linear story.

1 Key knowledge and worked examples

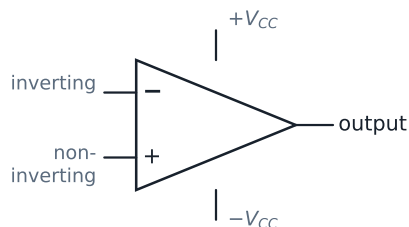
■ The ideal op-amp and its golden rules

An **operational amplifier** amplifies the difference between its + and - inputs. The **ideal operational amplifier** has infinite open-loop gain, infinite input resistance and zero output resistance. With **negative feedback** from output to the - input, two rules hold:

1. No current flows into either input.
2. The input difference is zero – a **virtual short** : the - input sits at whatever voltage the + input has.

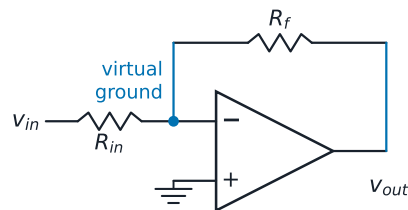
Saturation : the output cannot leave the supply rails (a ± 15 V op-amp stops near ± 13 – 14 V). Beyond that, rule 2 is false and linear answers are invalid.

Five terminals



ideal: infinite gain,
infinite input resistance

Inverting amplifier



the - node sits at 0 V, so
all of v_{in}/R_{in} flows through R_f

■ Worked example: the inverting amplifier

Input through R_{in} to the - node, feedback through R_f , + grounded. The - node is at 0 V (virtual short), so the input current v_{in}/R_{in} flows entirely through R_f :

$$\frac{v_{out}}{v_{in}} = -\frac{R_f}{R_{in}}$$

With $R_{in} = 10k\Omega$ and $R_f = 100k\Omega$: gain -10 , and $v_{in} = 0.2V$ gives $v_{out} = -2V$ – inside ± 15 V, so the linear model stands. The feedback current $0.2/10k = 20\mu A$ is supplied by the output, not the input.

■ Worked example: non-inverting stage and a saturation trap

Signal at +; feedback divider R_f over R_{in} at -. The - node must equal v_{in} , so $v_{out} = v_{in}(1 + R_f/R_{in})$ – always at least unity. With $R_{in} = 10k\Omega$, $R_f = 30k\Omega$: gain 4, and 0.5 V in gives 2 V out.

Now demand gain -100 ($R_f = 1M\Omega$, $R_{in} = 10k\Omega$) with $v_{in} = 0.3V$: the linear answer is -30 V, but ± 15 V rails stop the output near -14 V. The correct statement is “the output saturates”, not “the output is -30 V”.

2 Practice

2.1 An inverting amplifier has $R_{in} = 4.7k\Omega$ and $R_f = 47k\Omega$. State the gain, and the output for $v_{in} = +0.15V$. [3]

2.2 A non-inverting amplifier has $R_{in} = 10k\Omega$ (to ground) and $R_f = 30k\Omega$. State the gain and the output for $v_{in} = 0.5V$. [3]

2.3 An inverting amplifier with gain -100 receives $v_{in} = 0.3V$; the supplies are ± 15 V. Give the output the linear model predicts, the actual output, and the condition under which the linear model would be trustworthy. [3]

3 Exam-style questions

3.1 Design an inverting stage with gain -6 using $R_{in} = 10k\Omega$, stating the feedback resistor. For $v_{in} = 1.2V$ (peak), compute the output, check it against ± 15 V rails, and compute the feedback current. Then explain why the input source sees an input resistance of exactly R_{in} . [6]

3.2 Two cascaded inverting stages use ideal op-amps with ± 15 V supplies: stage 1 has $R_{in} = 10k\Omega$, $R_f = 20k\Omega$; stage 2 has $R_{in} = 15k\Omega$, $R_f = 45k\Omega$.

Compute each gain and the overall gain. For $v_{in} = 0.4V$, give the voltage at each stage output, verify neither stage saturates, and state why the virtual short applies to both stages. [6]

Solutions

2.1

- Gain: $-R_f/R_{in} = -47/4.7 = -10$.
- Output: $-10 \times 0.15 = -1.5V$.
- Inside $\pm 15\text{ V}$ – no saturation.

2.2

- Gain: $1 + R_f/R_{in} = 1 + 30/10 = 4$.
- Output: $4 \times 0.5 = 2V$.
- Positive because the signal enters the + input.

2.3

- Linear prediction: $-100 \times 0.3 = -30V$.
- Actual: saturated near -14 V (rail minus output-stage headroom).
- Trustworthy when $|gain \times v_{in}|$ stays a few volts inside the rails – here it doesn't.

3.1

- $R_f = 6 \times R_{in} = 60k\Omega$.
- Output: $-6 \times 1.2 = -7.2V$.
- Rail check: $7.2\text{ V} < 15\text{ V}$ – linear operation.
- Feedback current: $1.2/10k = 0.12mA$.
- The – node is a virtual ground (0 V), so the source drives R_{in} directly.
- Input resistance: source sees 0 V across $R_{in} \rightarrow$ input resistance is R_{in} by Ohm's law:
 $v_{in}/i_{in} = R_{in}$.

3.2

- Stage 1 gain: $-20/10 = -2$.
- Stage 2 gain: $-45/15 = -3$.
- Overall: $(-2) \times (-3) = +6$.
- Stage 1 output: $-2 \times 0.4 = -0.8V$; stage 2 output: $-3 \times (-0.8) = +2.4V$.
- Both magnitudes are far inside $\pm 15\text{ V}$ – no saturation.
- Each stage has its own feedback loop to its own – input, so the virtual short holds in both.