1.4 Source transformations and network theorems

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Thévenin's theorem • Norton's theorem • source transformation • superposition • maximum power transfer

Objective

Reduce a linear network to its Thévenin or Norton equivalent, apply superposition correctly (including knowing where it stops), and compute maximum power transfer with its efficiency limit.

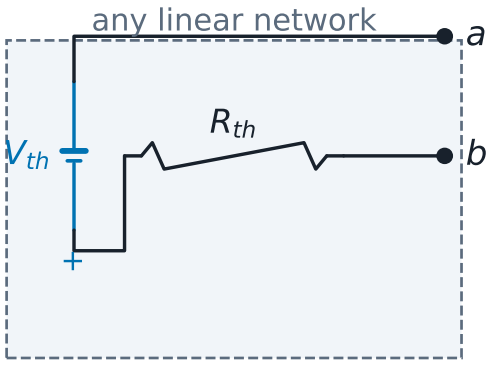
1 Key knowledge and worked examples

■ **Thévenin: the network becomes one source and one resistor**

Thévenin's theorem : any linear two-terminal network behaves, at its terminals, like a voltage source V_{th} in series with R_{th}. Find V_{th} as the open-circuit voltage; find R_{th} as the resistance seen from the terminals with independent sources zeroed (voltage sources shorted, current sources opened).

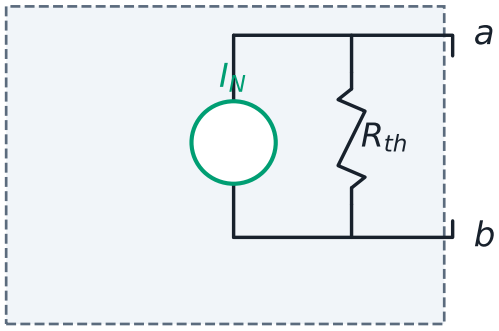
Norton's theorem is the same statement as $I_N = V_{th}/R_{th}$ in parallel with R_{th}; swapping between the two is the **source transformation** .

Thévenin: series form



measure: V_{th} open-circuit, R_{th} with sources zeroed

Norton: parallel form



swap the pair freely: $I_N = V_{th}/R_{th}$

■ Worked example: divider, then maximum power

A 10 V source with 4 Ω in series feeds a 6 Ω shunt; the load connects across the 6 Ω.

- $V_{th} = 10 \times 6/10 = 6V$ (open-circuit voltage across the shunt).
- $R_{th} = 4 \parallel 6 = 2.4\Omega$ (source shorted).
- With $R_L = R_{th} = 2.4\Omega$: $I = 6/4.8 = 1.25A$, $P_L = 1.25^2 \times 2.4 = 3.75W$, matching $P_{max} = V_{th}^2/(4R_{th}) = 36/9.6$.

At that match the source resistance burns another 3.75 W: **50% efficiency**. Matched loads belong in signal stages, not on the power grid.

■ Superposition: add responses, never powers

Superposition : with several independent sources, the total current or voltage is the sum of the responses to each source acting alone (others zeroed).

A 9 V source through 2 Ω feeds node A; a 1 A source pulls current out of A; an 8 Ω resistor goes from A to ground. Voltage source alone: $I = 9/10 = 0.9A$ in the 8 Ω. Current source alone: the 1 A leaves through 8 Ω ∥ 2 Ω, contributing $1 \times 2/10 = 0.2A$ out of the 8 Ω. Total: `0.9 - 0.2 = 0.7 A` down the 8 Ω. Nodal check: `4(V-9) + V + 8 = 0` (×8) gives $V = 5.6V$, and `5.6/8 = 0.7 A`. ✓

Power is quadratic – i^2R does not add across sub-solutions, so superposition never applies to power directly.

2 Practice

2.1 A 10 V source feeds 5 Ω in series with 20 Ω. For the terminals across the 20 Ω: compute V_{th} and R_{th}, then the short-circuit current I_N. [4]

2.2 Using the network of 2.1, find the load resistance for maximum power transfer and the maximum load power. [3]

2.3 In the panel example above (9 V, 2 Ω; 1 A out of A; 8 Ω to ground), compute the 8 Ω current caused by each source alone, then show that adding the two POWER values gives a different (wrong) answer from the true 8 Ω power. [4]

3 Exam-style questions

3.1 A 15 V source with 3 Ω in series feeds a 6 Ω shunt; a variable load R_L connects across the shunt terminals.

Find V_{th} and R_{th}. Then, for $R_L = R_{th}$, compute the load current, the load power, the power wasted in R_{th}, and the efficiency. Finally state, with one calculation, whether halving R_L to $R_{th}/2$ raises or lowers the load power. [7]

3.2 Two sources feed one node: a 6 V source through 3 Ω and a 3 V source through 3 Ω (both driving current into the node), with a 3 Ω resistor from the node to ground.

Use superposition to find the node voltage and the current in the grounded resistor; verify by direct nodal analysis. Then explain why the energy dissipated in the grounded resistor is NOT the sum of the energies the two sources would dissipate it with alone. [6]

Solutions

2.1

- $V_{th} = 10 \times 20/25 = 8V$.
- $R_{th} = 5 \parallel 20 = 100/25 = 4\Omega$.
- $I_N = V_{th}/R_{th} = 8/4 = 2A$.
- Check: short-circuit current from the full circuit: `10/5 = 2 A`. ✓

2.2

- Maximum power at $R_L = R_{th} = 4\Omega$.
- $P_{max} = V_{th}^2/(4R_{th}) = 64/16 = 4W$.
- Loop check: $I = 8/8 = 1A$; $P = 1^2 \times 4 = 4W$. ✓

2.3

- 9 V source alone: `0.9 A` down the 8 Ω; power $0.81 \times 8 = 6.48W$.
- 1 A source alone: `0.2 A` up (out of ground through the 8 Ω means 0.2 A flows upward); power $0.04 \times 8 = 0.32W$.
- Adding powers: `6.48 + 0.32 = 6.8 W` $\neq$ true power.
- True: total current `0.7 A`, power $0.49 \times 8 = 3.92W$ – cross terms are missing from the naive sum.

3.1

- $V_{th} = 15 \times 6/9 = 10V$.
- $R_{th} = 3 \parallel 6 = 2\Omega$.
- With $R_L = 2\Omega$: $I = 10/4 = 2.5A$.
- Load power: $2.5^2 \times 2 = 12.5W$; wasted in R_{th}: also 12.5 W.
- Efficiency: `12.5/25 = 50%`.
- Halving R_L to 1 Ω: $I = 10/3A$, $P_L = (10/3)^2 \times 1 \approx 11.1W < 12.5W$ – it lowers the load power (and the efficiency).

3.2

- 6 V alone: divider $3 \parallel 3 = 1.5\Omega$, $V' = 6 \times 1.5/4.5 = 2V$.
- 3 V alone: same structure, `V'' = 1 V`.
- Total $V = 3V$; grounded current `3/3 = 1 A`.
- Nodal check: `(V-6)/3 + (V-3)/3 + V/3 = 0` $\rightarrow V = 3V$. ✓
- Alone-dissipated energies: $2^2/3 = 1.33W$ and $1^2/3 = 0.33W$, summing to 1.67 W.
- True dissipation: $3^2/3 = 3W$ – superposition applies to V and i, not to v^2/R.