1.3 Nodal and mesh analysis

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS reference node • nodal analysis • mesh analysis • mesh current • supernode

Objective

Set up and solve node-voltage and mesh-current equations for resistive DC networks, choose the method with fewer unknowns, and verify every solution through an unused node or loop.

1 Key knowledge and worked examples

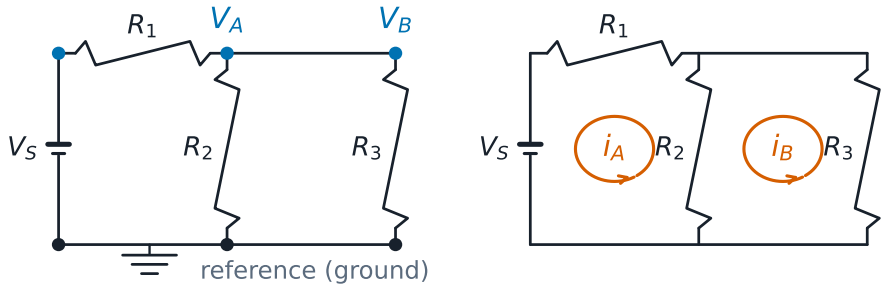
■ Nodal analysis: KCL at every unknown node

Choose a **reference node** (ground, 0 V), label the other node voltages, and write one KCL equation per node with branch currents as voltage differences over resistances. A voltage source fixed to ground removes an unknown; a current source fixes a branch current directly.

$$\frac{V_A - 12}{2} + \frac{V_A}{6} - 1 = 0 \;\Rightarrow\; 3(V_A - 12) + V_A - 6 = 0 \;\Rightarrow\; V_A = 10.5\ \text{V}$$

Check through the source branch: `(12 - 10.5)/2 = 0.75 A` in, plus 1 A in; the 6 Ω carries `10.5/6 = 1.75 A` out. The node balances.

Nodal: node voltages **Mesh: loop currents**



one KCL equation per unknown node — shared branch carries $i_A - i_B$

■ Worked example: mesh currents share a resistor

Two windows, mesh currents I_1 and I_2 both clockwise, shared 2 Ω resistor. Left loop: 12 V source and 1 Ω; right loop: no source and 2 Ω.

$$-12 + 1\,I_1 + 2(I_1 - I_2) = 0 \qquad 2(I_2 - I_1) + 2\,I_2 = 0$$

The first gives $3I_1 - 2I_2 = 12$; the second gives $I_1 = 2I_2$, so $I_2 = 3A$ and $I_1 = 6A$. The shared resistor carries $I_1 - I_2 = 3A$. Check loop 1: $-12 + 1 \times 6 + 2 \times 3 = 0$. ✓

Pick mesh when the circuit has current sources or few voltage sources; pick nodal when it has voltage sources or few current sources – count equations before starting.

■ Method choice is part of the answer

- A voltage source connected to ground fixes one node voltage outright: do not write an equation for it. A **supernode** handles a floating voltage source between two unknown nodes: wrap KCL around both and add the source constraint.
- Never mix directions: all mesh currents clockwise (or all counterclockwise) keeps the shared-resistor terms $R(I_1 - I_2)$ predictable.

2 Practice

2.1 A 12 V source connects through 4 Ω to node B; from B a 6 Ω resistor goes to ground. Using nodal analysis, find V_B, then verify it with the voltage-divider formula and check both branch currents at B. [4]

2.2 Write the two mesh equations (both currents clockwise) for a loop with a 9 V source and 2 Ω, sharing a 3 Ω resistor with a second loop containing 1 Ω and a 6 V source that drives current clockwise around loop 2 (so it aids I_2). Solve for both mesh currents. [5]

2.3 State, with a one-sentence reason for each, which method (nodal or mesh) gives fewer equations: (a) a four-node ladder with one grounded voltage source and five resistors; (b) a two-loop planar circuit whose only sources are two current sources. [3]

3 Exam-style questions

3.1 A 12 V source feeds node X through 2 Ω. Node X connects to node Y through 4 Ω, and Y connects to ground through 6 Ω. A 1 A current source pushes current into Y.

Write both node equations, solve for V_X and V_Y, and verify the answer by checking KCL at Y with all three branch currents evaluated numerically. [7]

3.2 A network has a 5 V source connected directly between two non-reference nodes P and Q (its + at P), with 2 Ω from P to ground, 3 Ω from Q to ground and 1 Ω from P to Q.

Explain why neither node equation can be written directly at P or Q, define the **supernode** equation and the constraint that accompanies it, and solve for V_P and V_Q. [6]

Solutions

2.1

- Equation: $(V_B - 12)/4 + V_B/6 = 0$.
- Multiply by 12: $3V_B - 36 + 2V_B = 0$, so $V_B = 7.2V$.
- Divider check: $12 \times 6/(4 + 6) = 7.2V$. ✓
- Currents: `(12-7.2)/4 = 1.2 A` in; `7.2/6 = 1.2 A` out – balanced.

2.2

- Loop 1: $-9 + 2I_1 + 3(I_1 - I_2) = 0 \rightarrow 5I_1 - 3I_2 = 9$.
- Loop 2 (6 V opposes the clockwise current in its 1 Ω): $-6 + 1I_2 + 3(I_2 - I_1) = 0 \rightarrow -3I_1 + 4I_2 = 6$.
- Solve: from the pair, $det = 20 - 9 = 11$.
- $I_1 = (9 \times 4 + 3 \times 6)/11 = 54/11 \approx 4.9A$.
- $I_2 = (5 \times 6 + 3 \times 9)/11 = 57/11 \approx 5.2A$.

2.3

- (a) Nodal: one source is grounded, so unknowns = nodes − 1 − 1 = 2, while the ladder has three meshes.
- (b) Mesh: each current source fixes one mesh current, leaving no mesh equations, while nodal has two unknown nodes.
- The rule: count unknowns for each method and take the smaller.

3.1

- Node X: $(V_X - 12)/2 + (V_X - V_Y)/4 = 0$.
- Node Y: $(V_Y - V_X)/4 + V_Y/6 - 1 = 0$.
- Clearing fractions: $3V_X - V_Y = 24$ and $-3V_X + 5V_Y = 12$.
- Adding: $4V_Y = 36 \rightarrow V_Y = 9V$; then $V_X = 11V$.
- Branch currents at Y: from X: `(11-9)/4 = 0.5 A` in; source: 1 A in; to ground: `9/6 = 1.5 A` out.
- `0.5 + 1 = 1.5` – KCL verified at Y.

3.2

- The 5 V source has no series resistance, so the branch current between P and Q cannot be written as a voltage difference over a resistance.
- Supernode KCL (P and Q together): $V_P/2 + V_Q/3 = 0$.
- Constraint: $V_P - V_Q = 5$.
- Substitute $V_P = V_Q + 5$: $(V_Q + 5)/2 + V_Q/3 = 0 \rightarrow 3V_Q + 15 + 2V_Q = 0$.
- $V_Q = -3V$, $V_P = 2V$.
- Power check: the source carries 1 A from Q to P and supplies 5 W; the shunts dissipate $2^2/2 = 2W$ and $3^2/3 = 3W$ – 5 W on each side.