Exercise walk-through

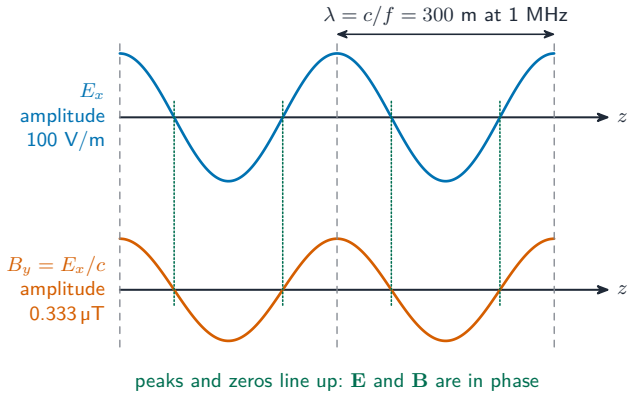
7.5
Plane waves

University Electrical Engineering

Plane waves: the trio and the impedance

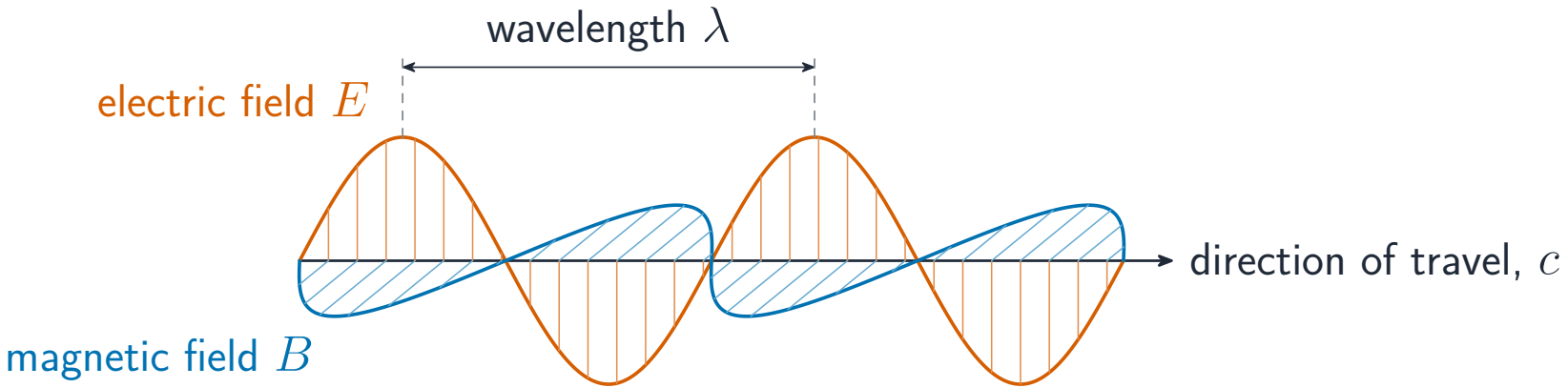
1. **Numbers** 100 V/m at 1 MHz: $B = 0.333$ uT, $\lambda = 300$ m, $S = 26.5$ W/m^2 - and $S = EB/\mu_0$ agrees.
2. **Boundary** Into ε_r 9: $\Gamma = -0.5$, SWR 3; reflected $0.25+$ transmitted $0.75 = 1$ - power conserved.
3. **Shield** 1 kHz into 1 mm aluminium: 0.37 skin depths, 31% attenuation; 12.4 mm needed for 100x.

E, B and 377 ohms



In phase, at right angles, B = E/c - and the power density S = E squared over 377.

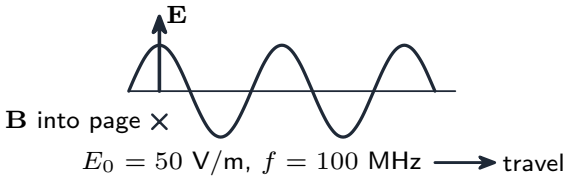
E and B in a plane wave



The wave travels at c, with $E = cB$

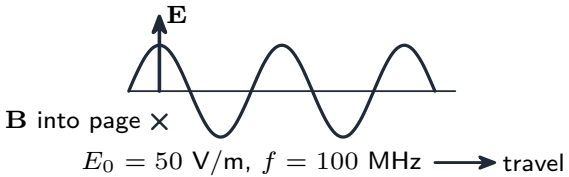
Exercise 2.1

2.1 A plane wave in air has a peak field $E_0 = 50$ V/m at 100 MHz. Find the peak B, the wavelength and the mean power density. [3]



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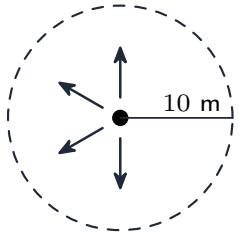


2.1

$$B_0 = \frac{E_0}{c} = \frac{50}{3.0 \times 10^8} = \mathbf{167\ nT};\ \lambda = \frac{c}{f} = \mathbf{3.0\ m};\ S = \frac{50^2}{2 \times 377} = \mathbf{3.3\ W/m^2}$$

Exercise 2.2

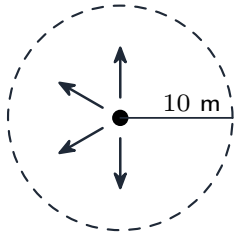
2.2 A 1.0 W transmitter radiates equally in all directions. Find the power density and the peak field at 10 m. [3]



1.0 W source at the centre

Exercise 2.2

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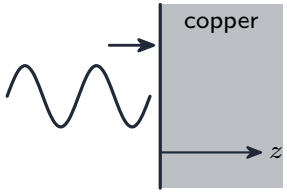
1.0 W source at the centre

2.2

$$S = \frac{P}{4\pi r^2} = \frac{1.0}{4\pi \times 10^2} = \mathbf{8.0 \times 10^{-4}\ W/m^2};\ E_0 = \sqrt{2\eta_0 S} = \mathbf{0.77\ V/m}$$

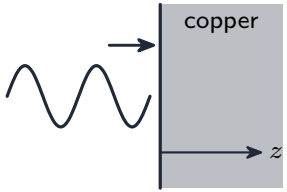
Exercise 2.3

2.3 Find the skin depth in copper ($\sigma = 5.8 \times 10^7$ S/m, $\mu = \mu_0$) at 10 MHz and at 3.0 GHz. [3]



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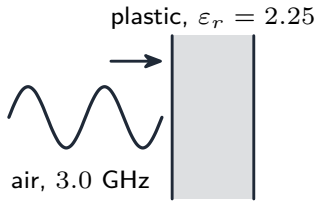


2.3

$\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}}$; at 10 MHz $\delta =$ **21 µm**; $\delta \propto \dfrac{1}{\sqrt{f}}$, so at 3.0 GHz it is $\dfrac{21}{\sqrt{300}} =$ **1.2 µm**

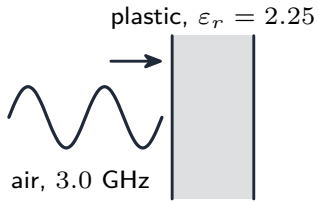
Exercise 2.4

2.4 A 3.0 GHz wave in air meets a plastic sheet with $\varepsilon_r = 2.25$, as shown. Find the wave impedance and the wavelength in the plastic, and the fraction of the power reflected at its front face. [3]



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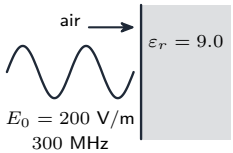
2.4

$\eta_2 = \dfrac{377}{1.5} = \mathbf{251}\ \Omega$; $\lambda = \dfrac{c}{f\sqrt{\varepsilon_r}} = \mathbf{6.7\ cm}$; $\Gamma = \dfrac{251 - 377}{251 + 377} = -0.20$, so $\mathbf{4}\%$ reflects

Exercise 3.1

3.1 A plane wave with $E_0 = 200$ V/m at 300 MHz in air hits a large flat lossless medium with $\varepsilon_r = 9.0$ and $\mu_r = 1$, at normal incidence.

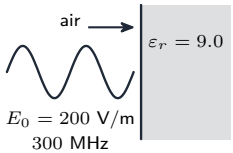
Find η in the medium, and Γ and τ. Find the reflected and transmitted fields, and the wavelength in the medium. Check that power is conserved. [5]



Exercise 3.1

3.1 A plane wave with $E_0 = 200$ V/m at 300 MHz in air hits a large flat lossless medium with $\varepsilon_r = 9.0$ and $\mu_r = 1$, at normal incidence.

Find η in the medium, and Γ and τ. Find the reflected and transmitted fields, and the wavelength in the medium. Check that power is conserved. [5]



3.1

$$\eta_2 = \frac{377}{3} = \mathbf{126}\ \Omega$$

$$\Gamma = \frac{126 - 377}{126 + 377} = \mathbf{-0.50},\ \tau = \mathbf{0.50}$$

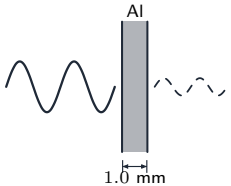
$$E_r = -100\ \text{V/m},\ E_t = \mathbf{100\ V/m};\ \lambda_2 = \frac{1.0\ \text{m}}{3} = \mathbf{0.33\ m}$$

$$S_i = \frac{200^2}{754} = 53.1\ \text{W/m}^2,\ S_r = 0.25 \times 53.1 = 13.3\ \text{W/m}^2,\ S_t = \frac{100^2}{2 \times 126} = 39.8\ \text{W/m}^2,$$
and $13.3 + 39.8 = 53.1$

Exercise 3.2

3.2 An aluminium shield ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$) is 1.0 mm thick, as shown.

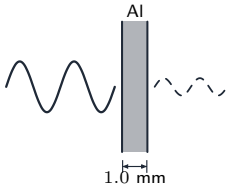
Find the skin depth at 1.0 kHz, and the loss of the field on one pass through the sheet, in dB. Repeat at 1.0 MHz. What does this tell you about using aluminium to shield a 1 kHz magnetic field? [5]



Exercise 3.2

3.2 An aluminium shield ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$) is 1.0 mm thick, as shown.

Find the skin depth at 1.0 kHz, and the loss of the field on one pass through the sheet, in dB. Repeat at 1.0 MHz. What does this tell you about using aluminium to shield a 1 kHz magnetic field? [5]



3.2

$$\delta = \frac{1}{\sqrt{\pi \times 10^3 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7}} = \mathbf{2.7\ mm}$$

$e^{-1.0/2.7} = 0.69$, a loss of only $8.69 \times \dfrac{1.0}{2.69} = \mathbf{3.2\ dB}$

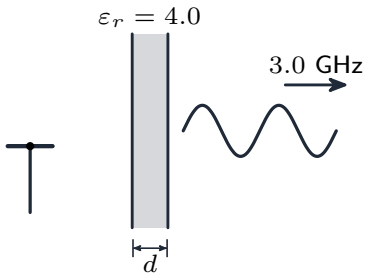
at 1.0 MHz $\delta = \dfrac{2.7}{\sqrt{1000}} = 85$ µm, so $\dfrac{t}{\delta} = 11.8$ and the loss is **102 dB**

aluminium blocks radio frequencies well but hardly touches a 1 kHz magnetic field; use a high-permeability material such as mu-metal or steel, whose large μ makes δ small

Exercise 3.3 (a)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

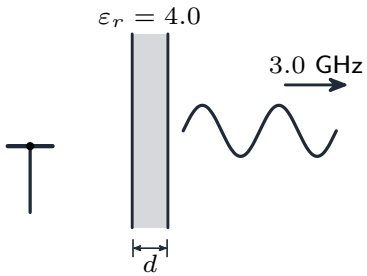
(a) For one air–dielectric face, find Γ, the fraction of the power reflected, and the loss in dB. [2]



Exercise 3.3 (a)

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(a) For one air–dielectric face, find Γ, the fraction of the power reflected, and the loss in dB. [2]



(a) $\Gamma = \dfrac{188.5 - 377}{188.5 + 377} = -\dfrac{1}{3}$, so **11**% reflects; loss $-10\log(1 - 0.11) = \mathbf{0.51\ dB}$

Exercise 3.3 (b)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(b) A sheet whose thickness is half a wavelength in the material lets the wave through with no reflection at its design frequency. Find this thickness at 3.0 GHz. [2]

Exercise 3.3 (b)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(b) A sheet whose thickness is half a wavelength in the material lets the wave through with no reflection at its design frequency. Find this thickness at 3.0 GHz. [2]

(b) $\lambda_d = \dfrac{c}{f\sqrt{\varepsilon_r}} = \dfrac{10\text{ cm}}{2} = 5.0\text{ cm};\ d = \dfrac{\lambda_d}{2} = \mathbf{2.5\ cm}$

Exercise 3.3 (c)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(c) The other choice is a very thin skin, about 1 mm. Give one advantage of each design. [2]

Exercise 3.3 (c)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(c) The other choice is a very thin skin, about 1 mm. Give one advantage of each design. [2]

(c) the half-wave sheet is strong and reflects almost nothing at the design frequency, but only near that frequency; the thin skin is light and works over a wide band, because it is too thin to reflect much at any frequency, but it is weaker

Exercise 3.3 (d)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(d) In rain, a water film ($\varepsilon_r \approx 80$) covers the radome. Find the power fraction one air–water face reflects, and say what this means for the radome's surface. [2]

Exercise 3.3 (d)

3.3 The 3 GHz link's antenna sits behind a flat **radome** (a protective cover) made of a dielectric with $\varepsilon_r = 4.0$, as shown.

(d) In rain, a water film ($\varepsilon_r \approx 80$) covers the radome. Find the power fraction one air–water face reflects, and say what this means for the radome's surface. [2]

(d) $\eta_w = \dfrac{377}{\sqrt{80}} = 42\ \Omega$, so $\Gamma = \dfrac{42 - 377}{42 + 377} = -0.80$; about **64**% of the power reflects: the surface must shed water (a water-repellent coating, or a sloped cover)