Exercise walk-through

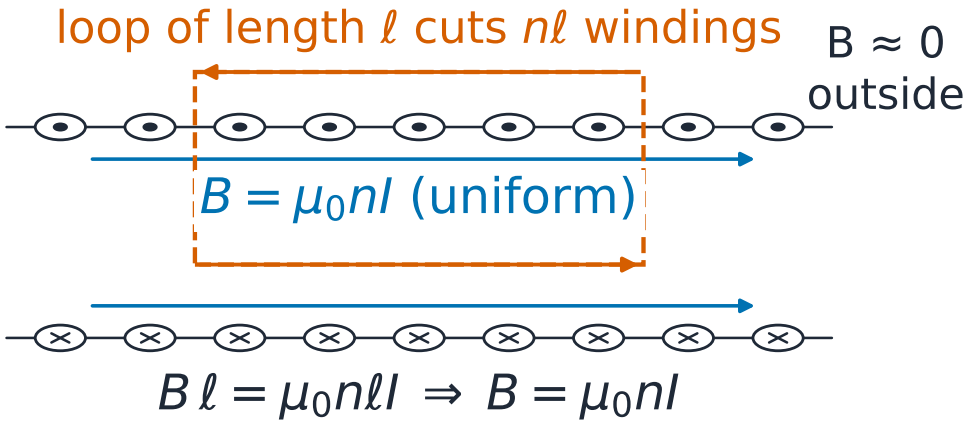
7.3 Magnetostatics

University Electrical Engineering

Magnetostatics: Ampere first, Biot-Savart when it fails

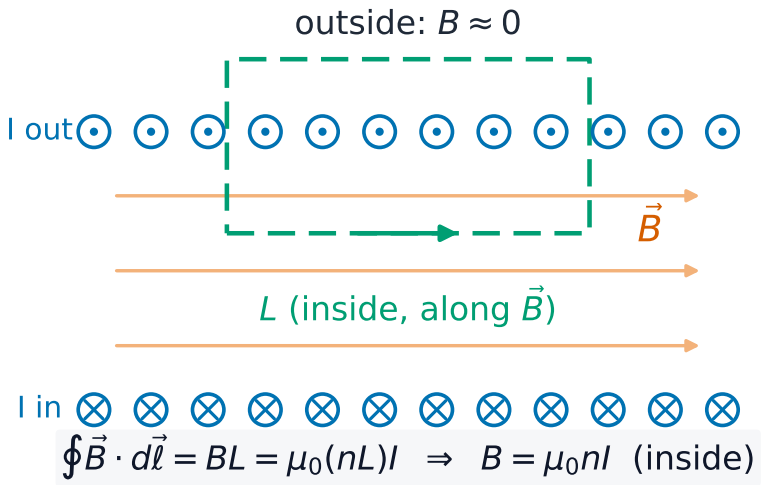
1. **Solenoid** 1000 turns/m at 2 A: B = 2.51 mT inside, ~0 outside.
2. **Orbit** A 1 uC charge at 10^6 m/s across 1 T feels 1 N; an electron circles at r = mv/qB = 5.7 um.
3. **Force** Loop near a wire: near side attracts 18 uN, far side repels 5.1; net toward the wire.

One line instead of a page



The rectangular Amperian loop cuts n-I windings: B = mu0 n I inside, zero outside.

The field inside a solenoid

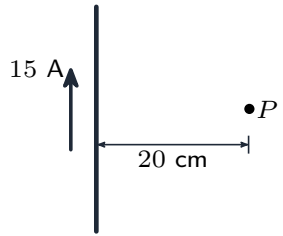


$$\oint \vec{B} \cdot d\vec{\ell} = BL = \mu_0 (nL) I \quad \Rightarrow \quad B = \mu_0 n I \quad \text{(inside)}$$

> The field inside is uniform and does not depend on the loop's position

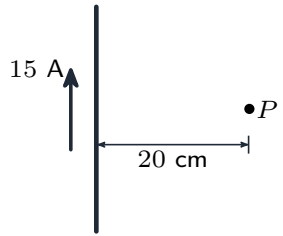
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2.1 A long straight wire carries 15 A. Find B at the point P shown, and compare it with the Earth's field of 50 µT. [3]



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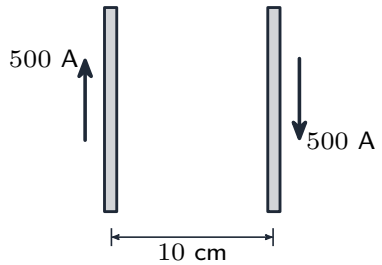


2.1

Ampère on a circle of radius r: $B \cdot 2\pi r = \mu_0 I$; $B = \dfrac{4\pi \times 10^{-7} \times 15}{2\pi \times 0.20} = \mathbf{15\ \mu T}$; about **30**% of the Earth's field, enough to upset a compass

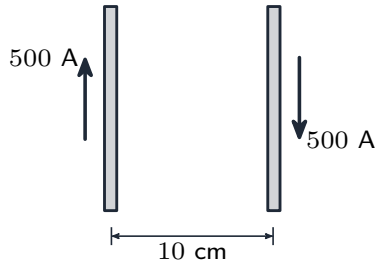
Exercise 2.2

2.2 Two long busbars 10 cm apart carry 500 A each, in opposite directions, as shown. Find the force on each metre of busbar, and say whether they attract or repel. [3]



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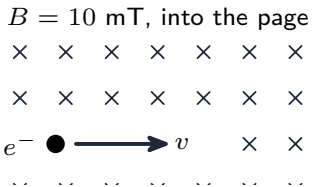


2.2

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{2 \times 10^{-7} \times 500 \times 500}{0.10} = \mathbf{0.50\ N/m}; \text{ opposite currents } \mathbf{repel}$$

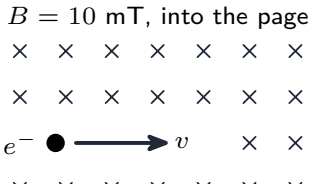
Exercise 2.3

2.3 An electron moves at 2.0×10^6 m/s at right angles to a uniform field $B = 10$ mT, as shown. Find the radius of its path and the time for one turn. ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C) [3]



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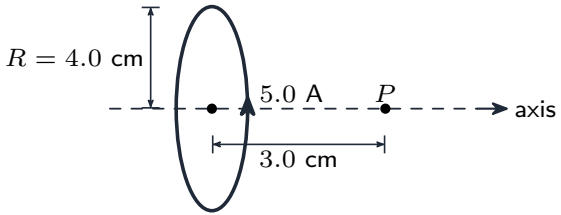


2.3

$$evB = \frac{mv^2}{r}, \text{ so } r = \frac{mv}{eB} = \frac{9.11 \times 10^{-31} \times 2.0 \times 10^6}{1.60 \times 10^{-19} \times 0.010} = \mathbf{1.14\ mm}; T = \frac{2\pi m}{eB} = \mathbf{3.6\ ns}$$

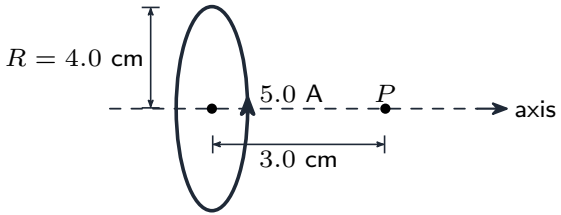
Exercise 2.4

2.4 A circular loop of radius $R = 4.0$ cm carries 5.0 A. Find B at its centre and at the point P on its axis, 3.0 cm from the centre. [3]



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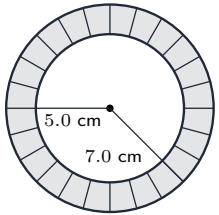
$$B(0) = \frac{\mu_0 I}{2R} = \frac{4\pi \times 10^{-7} \times 5.0}{0.080} = \mathbf{78.5\ \mu T};\ B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}} \text{ with } R^2 + z^2 = (5.0\text{ cm})^2;$$

$$78.5 \times \frac{4^3}{5^3} = \mathbf{40.2\ \mu T}$$

Exercise 3.1

3.1 The toroid shown has $N = 200$ turns and a square cross-section, 2.0 cm by 2.0 cm, from $r = 5.0$ cm to $r = 7.0$ cm. It carries $I = 1.5$ A.

Use Ampère's law to find B at the inner and outer radius. Find the flux in the core by integration, and the inductance. How far off is the answer if you use B at the mean radius? [5]

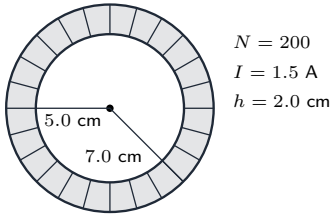


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3.1

a circle of radius r inside the core encloses NI: $B = \dfrac{\mu_0 NI}{2\pi r}$; $B = \mathbf{1.20\ mT}$ at 5.0 cm and **0.86 mT** at 7.0 cm

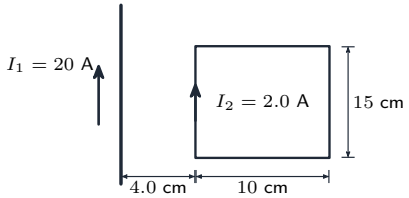
$\Phi = \int_a^b \dfrac{\mu_0 NI}{2\pi r} h\, dr = \dfrac{\mu_0 NIh}{2\pi} \ln \dfrac{b}{a} = 4.04 \times 10^{-7}$ Wb; $L = \dfrac{N\Phi}{I} = \mathbf{54\ \mu H}$

mean radius 6.0 cm: $B = 1.00$ mT and $L = 53.3$ µH, about 1% low, because the core is thin compared with its radius

Exercise 3.2

3.2 A long wire carries $I_1 = 20$ A. A rectangular loop carrying $I_2 = 2.0$ A lies in the same plane, as shown. Its 15 cm sides are parallel to the wire, at 4.0 cm and 14 cm from it.

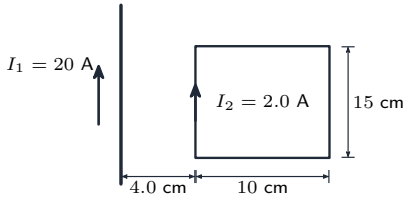
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3.2

near side: $\dfrac{2\times10^{-7}\times20\times2.0\times0.15}{0.040} = \mathbf{3.0\times10^{-5}\ N}$, towards the wire (like currents);

far side: $\dfrac{2\times10^{-7}\times20\times2.0\times0.15}{0.14} = \mathbf{8.6\times10^{-6}\ N}$, away

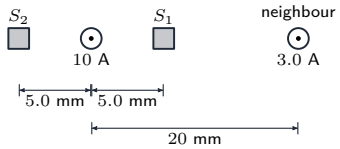
net force $\mathbf{2.1\times10^{-5}\ N}$ towards the wire; the short sides' forces cancel

$\Phi = \int_{0.04}^{0.14} \dfrac{\mu_0 I_1}{2\pi r}(0.15)\,dr = \dfrac{\mu_0 I_1(0.15)}{2\pi}\ln 3.5 = 7.5\times10^{-7}$ Wb; $M = \dfrac{\Phi}{I_1} = \mathbf{38\ nH}$

Setup for exercise 3.3

3.3 A **Hall sensor** measures the current in a long PCB trace from the field next to it. The figure is a cross-section. The trace carries up to 10 A. Sensor S_1 is 5.0 mm to its right and S_2 is 5.0 mm to its left. A neighbouring trace carries 3.0 A, 20 mm to the right of the main trace. Each sensor reads the field component along the same fixed direction.

(a) Find the field at S_1 from the main trace at 10 A. [2]



(b) Find the field at S_1 from the neighbouring trace, as a percentage of your answer to (a). [2]

(c) The design uses $S_1 - S_2$. Explain why this doubles the signal and removes the Earth's field. Find the error from the neighbour now, as a percentage. [2]

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Exercise 3.3

(d) A colleague suggests a closed magnetic ring round the trace instead. Explain, with Ampère's law, why this removes the neighbour's error, and give one limit of a simple ring sensor. [2]

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3.3

(a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{2 \times 10^{-7} \times 10}{0.0050} = \mathbf{400\ \mu T}$

(b) the neighbour is 15 mm from S_1: $\dfrac{2 \times 10^{-7} \times 3.0}{0.015} = 40\ \mu\text{T}$; **10**% of full scale

(c) the trace's field is opposite at S_1 and S_2, so $S_1 - S_2 = 800\ \mu\text{T}$, and a uniform field cancels; the neighbour gives $40 - 24 = 16\ \mu\text{T}$ (15 and 25 mm away), or **2**%

(d) the ring measures $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$; the neighbour is not enclosed, so it adds nothing; a plain ring (a current transformer) works only for ac, so dc needs a Hall element in a gap of the ring