Exercise walk-through

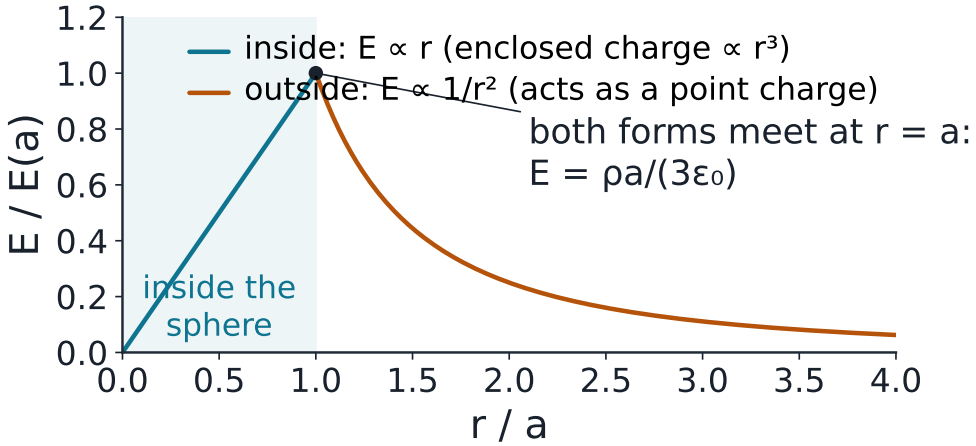
7.2
Electrostatics

University Electrical Engineering

Electrostatics: Gauss with boundary checks

1. **Sphere** Inside grows as r, outside falls as 1/r^2; the two agree at the boundary - the check that earns the marks.
2. **Dielectric** Fill all space with eps_r = 4: E falls 4x to 188 V/m at the surface; D is unchanged; bound charge -0.75 Q_free.
3. **Images** A charge near a grounded plane: mirror -q; induced total -q; force q^2/4 pi eps (2h)^2.

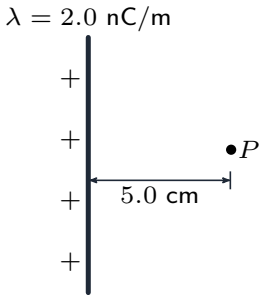
The sphere, both regions



E grows as r inside, falls as $1/r$ squared outside, and both formulas meet at the surface: 753 V/m.

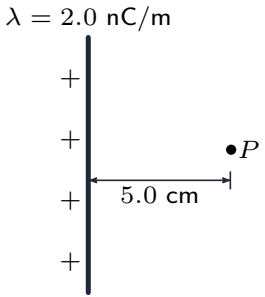
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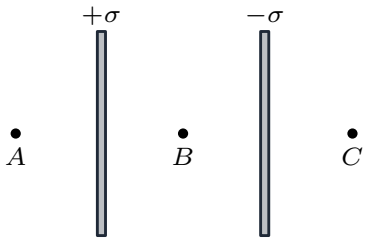
2.1

a coaxial cylinder of radius r and length L; no flux through its ends; $E \cdot 2\pi r L = \dfrac{\lambda L}{\varepsilon_0}$;

$$E = \frac{2.0 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12} \times 0.050} = \mathbf{719\ V/m}, \text{ away from the line}$$

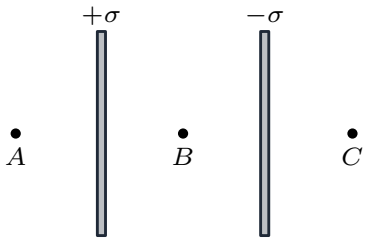
Exercise 2.2

2.2 Two large parallel sheets carry $+\sigma$ and $-\sigma$, with $\sigma = 5.0\ \mu\text{C/m}^2$. Find E at the points A, B and C shown. [3]



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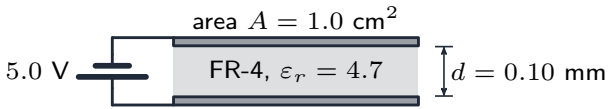


2.2

one sheet: $E = \dfrac{\sigma}{2\varepsilon_0} = 2.82 \times 10^5$ V/m; at B the two fields add: $\mathbf{5.65 \times 10^5\ V/m}$, from $+$ to $-$; at A and C they cancel: $\mathbf{0}$

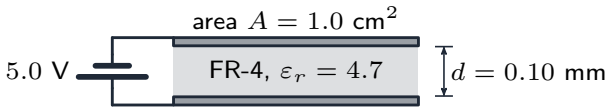
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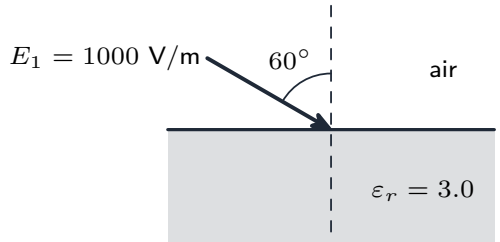


2.3

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{4.7 \times 8.854 \times 10^{-12} \times 1.0 \times 10^{-4}}{1.0 \times 10^{-4}} = \mathbf{41.6\ pF};\ W = \tfrac{1}{2} \times 41.6 \times 10^{-12} \times 5.0^2 = \mathbf{520\ pJ}$$

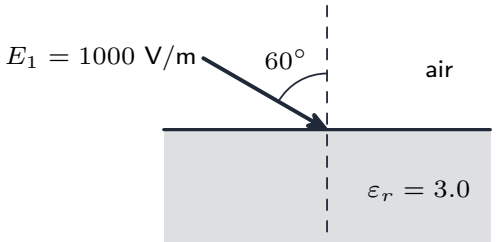
Exercise 2.4

2.4 In air, a field $E_1 = 1000$ V/m meets the flat surface of a dielectric with $\varepsilon_r = 3.0$, at 60° to the normal. There is no free charge on the surface. Find the size of the field E_2 inside and its angle to the normal. [3]



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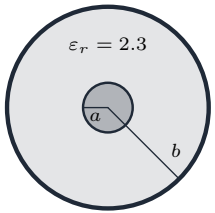
2.4

tangential part continuous: $E_{2t} = 1000 \sin 60° = 866$ V/m; normal D continuous: $E_{2n} = \dfrac{1000 \cos 60°}{3.0} = 167$ V/m; $E_2 = \sqrt{866^2 + 167^2} = \mathbf{882\ V/m}$ at $\tan^{-1} \dfrac{866}{167} = \mathbf{79°}$ to the normal

Exercise 3.1

3.1 The coaxial cable shown has inner radius $a = 1.0$ mm and outer radius $b = 4.0$ mm, filled with a dielectric of $\varepsilon_r = 2.3$. The inner conductor is at 100 V above the outer one.

Find $E(r)$ in terms of the charge per metre λ, and the capacitance per metre. Find the largest field in the cable, and say where it is. [5]

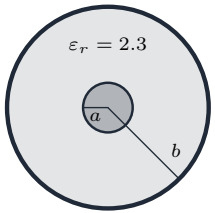


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3.1

Gauss on a coaxial cylinder: $E = \dfrac{\lambda}{2\pi\varepsilon r}$ for $a < r < b$

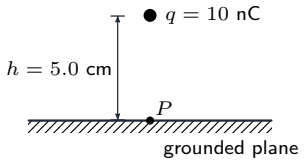
$V = \dfrac{\lambda}{2\pi\varepsilon}\ln\dfrac{b}{a}$; $C' = \dfrac{2\pi \times 2.3 \times 8.854 \times 10^{-12}}{\ln 4} = \mathbf{92.3\ pF/m}$

$E = \dfrac{V}{r\ln(b/a)}$ is largest at $r = a$: $\dfrac{100}{0.0010 \times \ln 4} = \mathbf{72\ kV/m}$; at the surface of the inner conductor, where breakdown starts first

Exercise 3.2

3.2 A point charge $q = 10$ nC is held $h = 5.0$ cm above a large grounded metal plane, as shown.

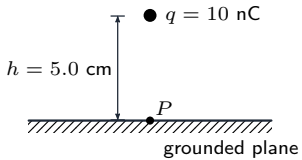
Use an image charge to find the field at P, and the surface charge density there. Find the force on the charge. [5]



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3.2

image $-q$ at 5.0 cm below the plane; the field above is that of q and $-q$

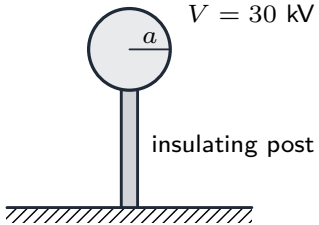
at P both fields point down: $E = 2 \times \dfrac{q}{4\pi\varepsilon_0 h^2} = 2 \times \dfrac{8.99 \times 10^9 \times 10 \times 10^{-9}}{0.050^2} = \mathbf{7.2 \times 10^4}$ $\mathbf{V/m}$, into the plane

$\sigma = -\varepsilon_0 E = \mathbf{-0.64\ \mu C/m^2}$; $F = \dfrac{q^2}{4\pi\varepsilon_0 (2h)^2} = \dfrac{8.99 \times 10^9 \times (10^{-8})^2}{0.10^2} = \mathbf{9.0 \times 10^{-5}}$ $\mathbf{N}$, towards the plane

Exercise 3.3 (a)

3.3 A test rig has a spherical high-voltage terminal of radius $a = 1.0$ cm on an insulating post, far from other objects. It is charged to $V = 30$ kV. Air breaks down at about 3.0 MV/m.

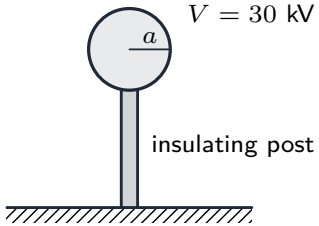
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Exercise 3.3 (a)

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(a) $V = \dfrac{Q}{4\pi\varepsilon_0 a}$ and $E = \dfrac{Q}{4\pi\varepsilon_0 a^2}$, so $E = \dfrac{V}{a}$; $E = \dfrac{30 \times 10^3}{0.010} = \mathbf{3.0\ MV/m}$: just at breakdown

Exercise 3.3 (b)

3.3 A test rig has a spherical high-voltage terminal of radius $a = 1.0$ cm on an insulating post, far from other objects. It is charged to $V = 30$ kV. Air breaks down at about 3.0 MV/m.

(b) Find the capacitance of the sphere and its charge. [2]

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(b) Find the capacitance of the sphere and its charge. [2]

(b) $C = 4\pi\varepsilon_0 a = \mathbf{1.11\ pF}$; $Q = CV = \mathbf{33\ nC}$

Exercise 3.3 (c)

3.3 A test rig has a spherical high-voltage terminal of radius $a = 1.0$ cm on an insulating post, far from other objects. It is charged to $V = 30$ kV. Air breaks down at about 3.0 MV/m.

(c) The design rule asks for a safety factor of 2 on the field. Find the smallest radius of sphere that meets it at 30 kV. [2]

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(c) need $E \le 1.5$ MV/m, so $a \ge \dfrac{30 \times 10^3}{1.5 \times 10^6} = \mathbf{2.0\ cm}$

Exercise 3.3 (d)

3.3 A test rig has a spherical high-voltage terminal of radius $a = 1.0$ cm on an insulating post, far from other objects. It is charged to $V = 30$ kV. Air breaks down at about 3.0 MV/m.

(d) The sphere is held on by a screw with a sharp head. Explain why this is a problem even with the right radius, and what to do about it. [2]

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(d) the field near a conductor goes roughly as V over the local radius of curvature, so a sharp edge has a much higher field and starts **corona** there; round off every edge, or hide the screw inside a smooth larger shape such as a corona ring