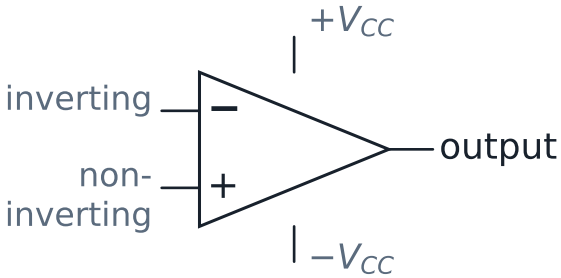
Exercise walk-through

1.6
Capacitors, inductors and energy

University Electrical Engineering

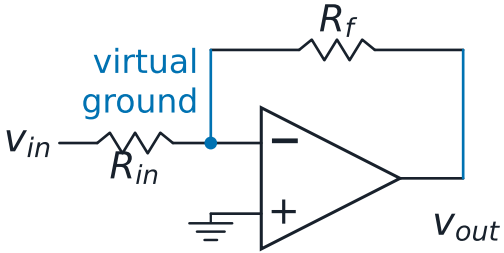
Five terminals and one taming trick

Five terminals



ideal: infinite gain,
infinite input resistance

Inverting amplifier

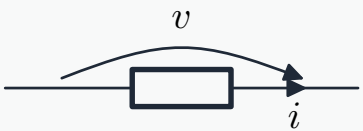


the − node sits at 0 V, so
all of v_{in}/R_{in} flows through R_f

The triangle has two inputs, one output and two supply rails; **negative feedback** pins the minus node to the plus node - the **virtual ground** that makes gain exact.

Burn, or store in a field

Resistor

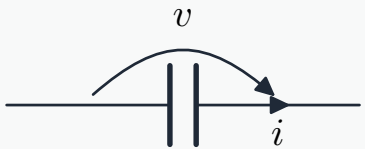


$$v = Ri$$

turns energy into heat,
$p = i^2 R$

stores nothing

Capacitor

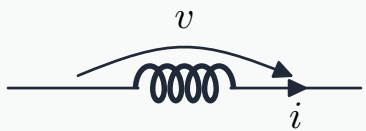


$$i = C\,\frac{dv}{dt}$$

electric field,
$w = \frac{1}{2}Cv^2$

holds its voltage

Inductor



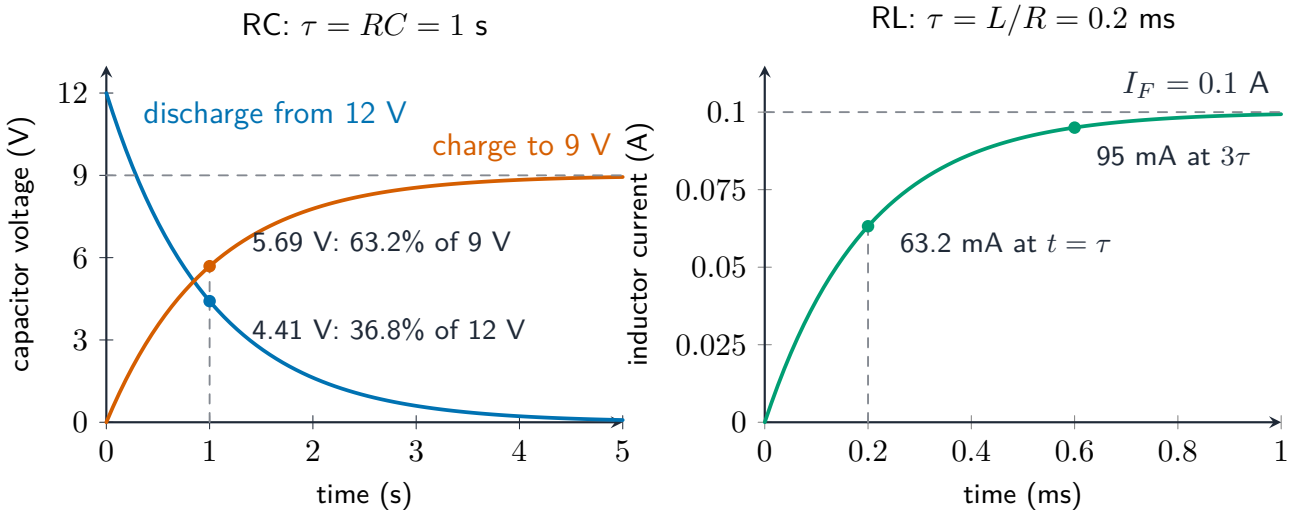
$$v = L\,\frac{di}{dt}$$

magnetic field,
$w = \frac{1}{2}Li^2$

holds its current

R burns energy; C and L store it, and so remember v and i.

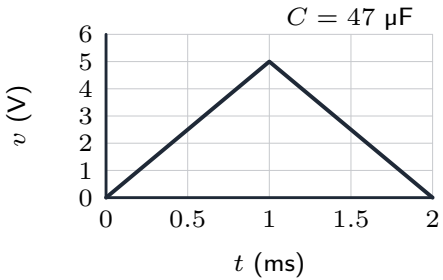
The same shape, capacitor or inductor



Charge and discharge share one curve family; τ marks the 63.2% point.

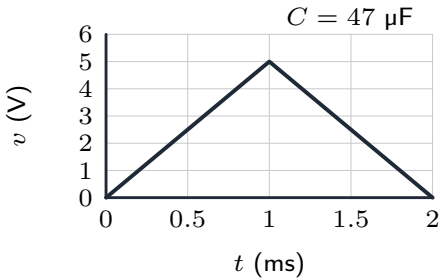
Exercise 2.1

2.1 The voltage on a 47 µF capacitor rises steadily from 0 to 5.0 V in 1.0 ms, then falls steadily back to 0 in the next 1.0 ms. Find the current in each part, and the largest stored energy. [3]



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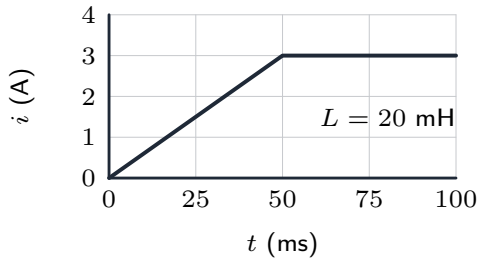


2.1

0–1 ms: $i = 47\times10^{-6}\times5000 = +\mathbf{0.235\ A}$; 1–2 ms: $-\mathbf{0.235\ A}$; $w_{\max} = \frac{1}{2}(47\times10^{-6})(5.0)^2 = \mathbf{0.59\ mJ}$

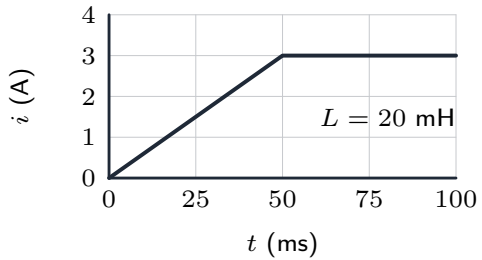
Exercise 2.2

2.2 The current in a 20 mH inductor rises steadily from 0 to 3.0 A in 50 ms and then stays at 3.0 A. Find the voltage across it during the rise and after it, and the final stored energy. [3]



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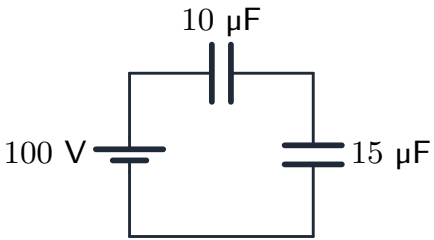


2.2

during the rise: $v = 0.020 \times \dfrac{3.0}{0.050} = \mathbf{1.2\ V}$; after: $\dfrac{di}{dt} = 0$, so $v = \mathbf{0}$; $w = \tfrac{1}{2}(0.020)(3.0)^2 = \mathbf{90\ mJ}$

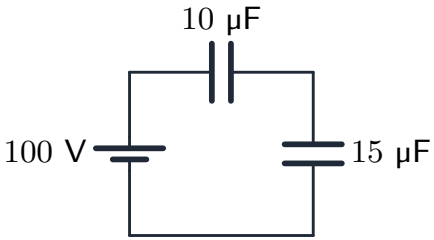
Exercise 2.3

2.3 Capacitors of 10 µF and 15 µF are in series across 100 V. Find the total capacitance, the charge on each, and the voltage across each. [3]



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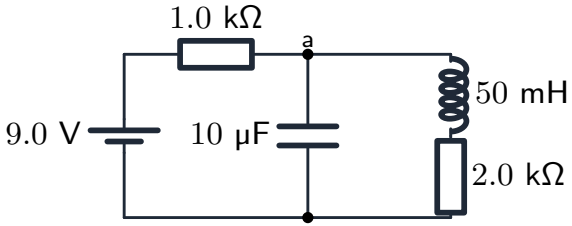


2.3

$C = \dfrac{10 \times 15}{25} = \mathbf{6.0\ \mu F}$; $q = 6.0 \times 10^{-6} \times 100 = \mathbf{600\ \mu C}$ on each; $v_{10} = \dfrac{600}{10} = \mathbf{60\ V}$, $v_{15} = \mathbf{40\ V}$

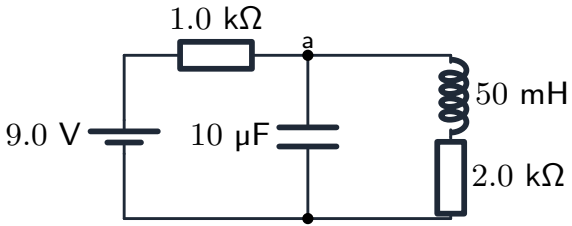
Exercise 2.4

2.4 A 9.0 V source feeds node a through 1.0 kΩ. From a, a 10 µF capacitor goes to ground, and a 50 mH inductor in series with 2.0 kΩ also goes to ground. Find v_C, i_L and both stored energies after a long time. [3]



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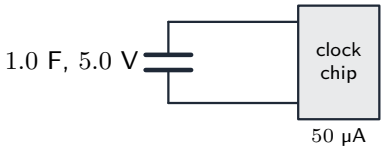
2.4

C open, L short: $i_L = \dfrac{9.0}{3.0\text{ k}} = \mathbf{3.0\ mA}$; $v_C = 3.0\text{ mA} \times 2.0\text{ k}\Omega = \mathbf{6.0\ V}$; $w_C = \frac{1}{2}(10^{-5})(36) = \mathbf{0.18\ mJ}$, $w_L = \frac{1}{2}(0.050)(0.0030)^2 = \mathbf{0.23\ \mu J}$

Exercise 3.1

3.1 A real-time clock is kept running by a 1.0 F **supercapacitor** when the main power fails. The supercapacitor starts at 5.0 V. The clock draws a steady 50 µA and stops working below 2.0 V.

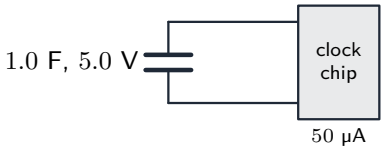
Find how long the clock keeps running and the energy the supercapacitor gives out. Check the energy another way. The clock must survive a 24-hour power cut: what capacitance is needed? [5]



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Find how long the clock keeps running and the energy the supercapacitor gives out. Check the energy another way. The clock must survive a 24-hour power cut: what capacitance is needed? [5]



3.1

constant current, so $\Delta v = \dfrac{i\,t}{C}$: $t = \dfrac{1.0 \times 3.0}{50 \times 10^{-6}} = 6.0 \times 10^{4}$ s = **16.7 h**

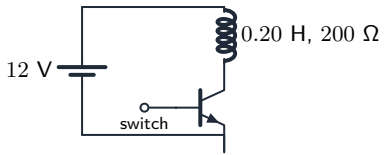
energy $= \frac{1}{2}(1.0)(5.0^2 - 2.0^2) =$ **10.5 J**; check: average voltage 3.5 V $\times$ 50 µA $\times 6.0 \times 10^{4}$ s $= 10.5$ J

$C = \dfrac{50 \times 10^{-6} \times 86\,400}{3.0} = 1.44$ F, so use a **1.5 F** part

Exercise 3.2

3.2 A transistor switches a relay coil ($L = 0.20$ H, resistance 200 Ω) from a 12 V supply. The transistor turns off in about 1.0 µs.

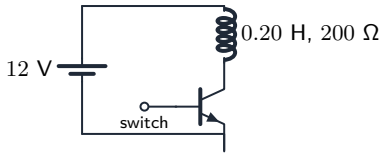
Find the coil current and its stored energy, and estimate the voltage across the coil when the transistor turns off. Explain the danger, and how a diode across the coil removes it. [5]



Exercise 3.2

3.2 A transistor switches a relay coil ($L = 0.20$ H, resistance 200 Ω) from a 12 V supply. The transistor turns off in about 1.0 µs.

Find the coil current and its stored energy, and estimate the voltage across the coil when the transistor turns off. Explain the danger, and how a diode across the coil removes it. [5]



3.2

$i = \dfrac{12}{200} = 60$ mA, $w = \frac{1}{2}(0.20)(0.060)^2 = \mathbf{0.36\ mJ}$

$v = L\dfrac{di}{dt} \approx 0.20 \times \dfrac{0.060}{1.0 \times 10^{-6}} = \mathbf{12\ kV}$

the current in L cannot stop at once; the huge voltage breaks down the transistor or sparks

a **flyback diode** across the coil gives the current a path: it decays with $\tau = \dfrac{L}{R} = 1.0$ ms, and the transistor sees only about 12.7 V

Exercise 3.3 (a)

3.3 A camera flash stores energy in a 330 µF capacitor charged to 300 V. A converter charges it from a 3.0 V battery that can give 1.5 A; the converter is 80% efficient.

(a) Find the stored energy and the charge. [2]

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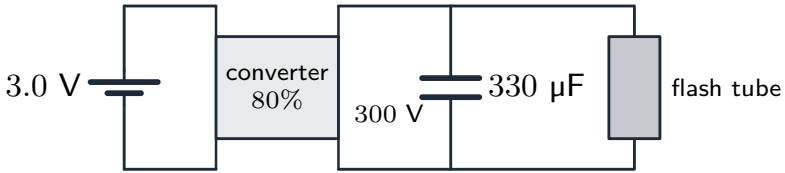
(a) Find the stored energy and the charge. [2]

(a) $w = \frac{1}{2}(330 \times 10^{-6})(300)^2 = \mathbf{14.9\ J}$; $q = Cv = \mathbf{0.099\ C}$

Exercise 3.3 (b)

3.3 A camera flash stores energy in a 330 µF capacitor charged to 300 V. A converter charges it from a 3.0 V battery that can give 1.5 A; the converter is 80% efficient.

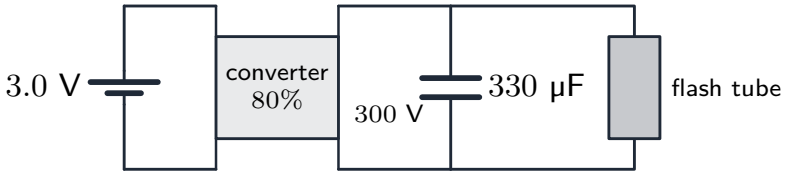
(b) The flash empties the capacitor in 1.0 ms. Find the average power in the flash. Find the shortest time to recharge. [3]



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(b) $P = \dfrac{14.85}{1.0 \times 10^{-3}} = \mathbf{14.9\ kW}$; power into C: $0.80 \times 3.0 \times 1.5 = 3.6$ W; $t = \dfrac{14.85}{3.6} = \mathbf{4.1\ s}$

Exercise 3.3 (c)

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(c) The product must recharge in 3.0 s. Give two ways to meet this, with numbers, and say which you would choose. [3]

Exercise 3.3 (c)

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(c) The product must recharge in 3.0 s. Give two ways to meet this, with numbers, and say which you would choose. [3]

(c) more current: $\dfrac{14.85}{3.0 \times 0.80 \times 3.0} = \mathbf{2.1\ A}$ from the battery; or less energy: $3.6 \times 3.0 =$ 10.8 J, so $C = \dfrac{2 \times 10.8}{300^2} = \mathbf{240\ \mu F}$

a choice with a reason: e.g. 2.1 A keeps the full flash if the battery can give it; 240 µF gives a 27% weaker flash but keeps the battery current low