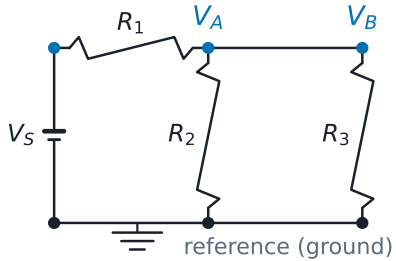
Exercise walk-through

1.3
Nodal and mesh analysis

University Electrical Engineering

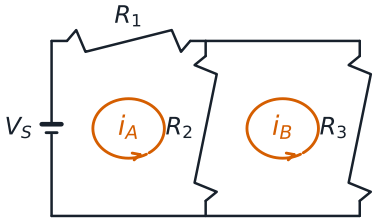
Nodal labels or mesh loops: same circuit, two labellings

Nodal: node voltages



one KCL equation per unknown node

Mesh: loop currents

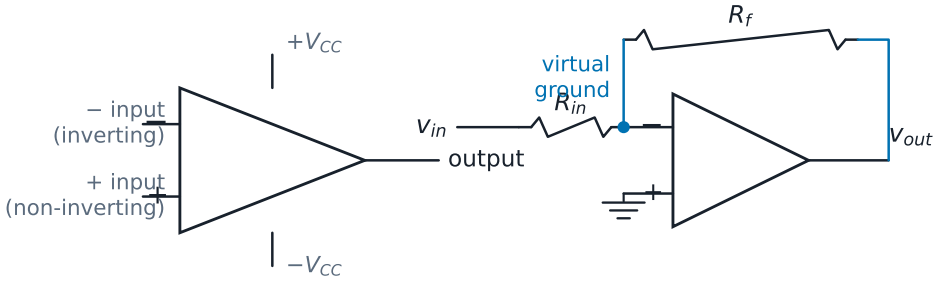


shared branch carries $i_A - i_B$

Left: unknown node voltages above a grounded reference. Right: circulating mesh currents whose difference crosses the shared branch.

Five terminals and one taming trick

The symbol: five terminals **The inverting amplifier**

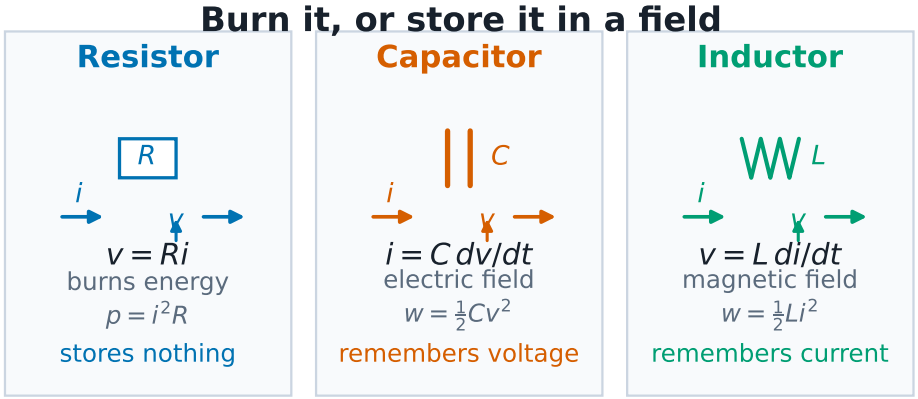


ideal model: infinite gain, infinite input resistance

rule 2 pins the − node at 0 V; rule 1 sends all of v_{in}/R_{in} through R_f

The triangle has two inputs, one output and two supply rails; **negative feedback** pins the minus node to the plus node - the **virtual ground** that makes gain exact.

Burn, or store in a field



The **resistor** burns what it receives; the **capacitor** stores energy in an electric field and remembers its voltage; the **inductor** stores it in a magnetic field and remembers its current.

Exercise 2.1

2.1 A 12 V source connects through 4 Ω to node B; from B a 6 Ω resistor goes to ground. Using nodal analysis, find V_B, then verify it with the voltage-divider formula and check both branch currents at B. [4]

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2.1

- Equation: $(V_B - 12)/4 + V_B/6 = 0$.
- Multiply by 12: $3V_B - 36 + 2V_B = 0$, so $V_B = 7.2V$.
- Divider check: $12 \times 6/(4+6) = 7.2V$. ✓
- Currents: `(12-7.2)/4 = 1.2 A` in; `7.2/6 = 1.2 A` out – balanced.

Exercise 2.2

2.2 Write the two mesh equations (both currents clockwise) for a loop with a 9 V source and 2 Ω, sharing a 3 Ω resistor with a second loop containing 1 Ω and a 6 V source that drives current clockwise around loop 2 (so it aids I_2). Solve for both mesh currents. [5]

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2.2

- Loop 1: $-9 + 2I_1 + 3(I_1 - I_2) = 0 \rightarrow 5I_1 - 3I_2 = 9$.
- Loop 2 (6 V opposes the clockwise current in its 1 Ω): $-6 + 1I_2 + 3(I_2 - I_1) = 0 \rightarrow -3I_1 + 4I_2 = 6$.
- Solve: from the pair, $det = 20 - 9 = 11$.
- $I_1 = (9 \times 4 + 3 \times 6)/11 = 54/11 \approx 4.9A$.
- $I_2 = (5 \times 6 + 3 \times 9)/11 = 57/11 \approx 5.2A$.

Exercise 2.3

2.3 State, with a one-sentence reason for each, which method (nodal or mesh) gives fewer equations: (a) a four-node ladder with one grounded voltage source and five resistors; (b) a two-loop planar circuit whose only sources are two current sources. [3]

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2.3

- (a) Nodal: one source is grounded, so unknowns = nodes − 1 − 1 = 2, while the ladder has three meshes.
- (b) Mesh: each current source fixes one mesh current, leaving no mesh equations, while nodal has two unknown nodes.
- The rule: count unknowns for each method and take the smaller.

Exercise 3.1

3.1 A 12 V source feeds node X through 2 Ω. Node X connects to node Y through 4 Ω, and Y connects to ground through 6 Ω. A 1 A current source pushes current into Y.

Write both node equations, solve for V_X and V_Y, and verify the answer by checking KCL at Y with all three branch currents evaluated numerically. [7]

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- Node X: $(V_X - 12)/2 + (V_X - V_Y)/4 = 0$.
- Node Y: $(V_Y - V_X)/4 + V_Y/6 - 1 = 0$.
- Clearing fractions: $3V_X - V_Y = 24$ and $-3V_X + 5V_Y = 12$.
- Adding: $4V_Y = 36 \rightarrow V_Y = 9V$; then $V_X = 11V$.
- Branch currents at Y: from X: `(11-9)/4 = 0.5 A` in; source: 1 A in; to ground: `9/6 = 1.5 A` out.
- `0.5 + 1 = 1.5` – KCL verified at Y.

Exercise 3.2

3.2 A network has a 5 V source connected directly between two non-reference nodes P and Q (its + at P), with 2 Ω from P to ground, 3 Ω from Q to ground and 1 Ω from P to Q.

Explain why neither node equation can be written directly at P or Q, define the **supernode** equation and the constraint that accompanies it, and solve for V_P and V_Q. [6]

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3.2

- The 5 V source has no series resistance, so the branch current between P and Q cannot be written as a voltage difference over a resistance.
- Supernode KCL (P and Q together): $V_P/2 + V_Q/3 = 0$.
- Constraint: $V_P - V_Q = 5$.
- Substitute $V_P = V_Q + 5$: $(V_Q + 5)/2 + V_Q/3 = 0 \rightarrow 3V_Q + 15 + 2V_Q = 0$.
- $V_Q = -3V$, $V_P = 2V$.
- Power check: the source carries 1 A from Q to P and supplies 5 W; the shunts dissipate $2^2/2 = 2W$ and $3^2/3 = 3W$ – 5 W on each side.