Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\log n < \sqrt{n} < n$

$< n \log n < n^2$

$< 2^n$ last

2.2

$i = 1, 2, 4, \ldots, 512$: **10** times

$\Theta(\log n)$

2.3

for $n \geq 1$: $5n \leq 5n^2$ and $2 \leq 2n^2$

so $3n^2 + 5n + 2 \leq 3n^2 + 5n^2 + 2n^2 = 10n^2$

$c = 10$, $n_0 = 1$

2.4

n doubles, so the time grows by $2^3 = 8$

$8 \times 8 =$ **64** s

3.1

(a) `2`: the pairs (3, 1) and (3, 2); the number of pairs that are out of order (inversions)

(b) $10 \times 9/2 =$ **45**; $n(n-1)/2$

(c) $\Theta(n^2)$: the test runs $n(n-1)/2$ times whatever the data

(d) a list in decreasing order, e.g. `[5, 4, 3, 2, 1]`; every pair is out of order: $5 \times 4/2 =$ **10**

3.2

(a) growing from 1, 2 and 4 copies $1 + 2 + 4 =$ **7**

(b) the copies are $1 + 2 + \cdots + 2^{k-1} = 2^k - 1 = n - 1 < n$; with the n writes, the total is under $2n$, so $O(1)$ per append on average

(c) about $n/10$ copies of up to n elements: $\Theta(n^2)$

3.3

(a) A: $100 \times 1000 \times 10 = 10^6$ ns = 1 ms; B: 10^6 ns = 1 ms, the same

(b) A: $100 \times 10^6 \times 20 = 2 \times 10^9$ ns = **2 s**; B: 10^{12} ns = **1000 s**

(c) A: $100 \times 100 \times 6.64 \approx 66\,400$ ns; B: 10 000 ns, so B is faster for small lists

(d) A, or a mix that uses B below about 1000 items: B saves only microseconds on small lists, but its 1000 s on the rare large list is far too slow

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