Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

5/36

$6/36 = 1/6$

$6/36 = 1/6$

2.2

$1 - (5/6)^4$

$= \mathbf{0.518}$

2.3

three outcomes are left: HH, HT, TH

1/3

2.4

$E = \frac{1}{6} \times 4 - 1$

$= -£0.33$ per game

over 60 games: $-£20$

3.1

(a) $0.2 \times 0.3 + 0.8 \times 0.01 = \mathbf{0.068}$

(b) $0.06/0.068 = \mathbf{0.882}$

(c) $8000 \times 0.01 = \mathbf{80}$ good e-mails blocked; $2000 \times 0.3 = \mathbf{600}$ spam caught

(d) no: 80 lost good e-mails is too many; combine it with other evidence, or send to a spam folder instead

3.2

(a) $1 - \prod_{i=0}^{22}(1 - i/365) = \mathbf{0.507}$

(b) exact: $\prod_{i=0}^{39}(1 - i/1000) = 0.454$, so $P(\text{collision}) = \mathbf{0.546}$; approximation $1 - e^{-40\times39/2000} = 1 - 0.458 = 0.542$

3.3

(a) $0.05^3 = \mathbf{1.25 \times 10^{-4}}$

(b) the 2nd try happens with probability 0.05, the 3rd with 0.05^2; $E = 1 + 0.05 + 0.0025$ $= \mathbf{1.0525}$

(c) the tries are no longer independent: if the first fails because the server is down, quick retries also fail, so the real chance is much higher than 1.25×10^{-4}

exercises.lol exercises.com.cn

(d) $0.05^k < 10^{-6}$ needs $k > 4.6$, so **5** tries; wait longer between tries (back-off), so a short outage can end