Solutions

2.1

- $9 - 1 = 8$ ✓.

2.2

- Sum 10, so $|E| = 5$ ✓.

2.3

- No – the degree sum must be even, so odd degrees come in pairs ✓.

3.1

- Total $4 + 4 + 3 + 3 + 3 + 2 + 2 = 21$ – odd, violating handshake ✓.
- Change one 2 to a 3: total $22 = 2 \times 11$ edges ✓.
- Corollary: the count of odd-degree vertices is even – the original list had three odd entries (impossible) ✓.
- Any drawing must satisfy handshake; the arithmetic screens drawings before pencil moves ✓.

3.2

- $|E| = 10 - 1 = 9$; degree sum $= 2 \times 9 = 18$ ✓.
- Internal sum: $18 - 4 = 14$ across 6 vertices ✓.
- Minimum internal degree 2 (path through) ✓.
- Let x degree-2 and $6 - x$ degree-3: $2x + 18 - 3x = 14 \Rightarrow x = 4$ ✓.