Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$10^4 = \mathbf{10{,}000}$

$10 \times 9 \times 8 \times 7 = \mathbf{5040}$

$10{,}000 - 5040 = \mathbf{4960}$

2.2

$\binom{10}{3} = \mathbf{120}$

$P(10, 3) = 10 \times 9 \times 8 = \mathbf{720}$

2.3

$62^8 \approx \mathbf{2.18 \times 10^{14}}$

$2.18 \times 10^{14} / 10^{10} = 21{,}800$ s

$\approx \mathbf{6.1}$ hours

2.4

there are 366 possible birthdays, including 29 February

367 people in 366 boxes: by the pigeonhole principle two share a day

3.1

(a) 8 steps, choose which 3 are up: $\binom{8}{3} = \mathbf{56}$

(b) to $(2, 1)$: $\binom{3}{1} = 3$; from $(2, 1)$ to $(5, 3)$: $\binom{5}{2} = 10$; $3 \times 10 = \mathbf{30}$

(c) $56 - 30 = \mathbf{26}$

(d) $30/56 = \mathbf{0.536}$

3.2

(a) $2^8 = \mathbf{256}$

(b) $\binom{8}{3} = \mathbf{56}$

(c) $256 - 1 = \mathbf{255}$

(d) $2^7 + 2^6 - 2^5 = 128 + 64 - 32 = \mathbf{160}$

3.3

(a) $26^8 \approx 2.09 \times 10^{11}$: about **21 s**

(b) all 8-character strings minus those with no digit: $62^8 - 52^8 \approx 1.65 \times 10^{14}$; about 16,500 s $\approx$ **4.6 hours**

(c) $26^{12} \approx 9.5 \times 10^{16}$: about 9.5×10^6 s $\approx$ **110 days**

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(d) C: length grows the count much faster than a forced digit, and long passwords are easier to remember; e.g. store passwords with a slow hash, or limit guesses